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NUMERICAL EXPERIENCE WITH TWO-POINT ADAPTIVE NONLINEARITY APPROXIMATION FOR DESIGN OPTIMIZATION

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1. Introduction

During the last decade numerous approximation techniques have taken place for application in optimization. Therefore, function approximation techniques emerge as one of the most important fields of interest. A thorough survey on this topic can be found in [Barthelemy and Haftka 1993].

The main objective of various approximation techniques in design and structural optimization is the reduction of repetitive evaluation of cost and constraint functions. In general, more precise function approximation provides faster convergence and more efficient achievement of optimum design. Consequently, the quality of function approximations emerge as one of the most important issues in the field of design and structural optimization.

The main purpose of this paper is to investigate and quantify the accuracy of the Two-Point Adaptive Nonlinearity Approximation, *TANA-3* [Xu and Grandhi 1998]. The result could be a valuable tool in comparison with other approximation schemes such as the Generalized Convex Approximation, *GCA* [Chickermane and Gea 1996], numerically evaluated in [Magazinović 2000].

The outline of the paper is as follows: In Section 2, a brief overview of the Two-Point Adaptive Nonlinearity Approximation algorithm is presented. In Section 3, a description of performed numerical tests is given and finally, in Section 4, numerical results are presented.

2. Two-Point Adaptive Nonlinearity Approximation

2.1 TANA-3 Basics

The Two-Point Adaptive Nonlinearity Approximation, *TANA-3*, is the last and probably the best approximation scheme in a well known series of four function approximations: *TANA* [Wang and Grandhi 1994], *TANA-1* and *TANA-2* [Wang and Grandhi 1995], and finally *TANA-3* [Xu and Grandhi 1998]. It is based on the function incomplete second order Taylor series expansion with respect to the intervening variables y_i

$$y_i = x_i^{p_i}, \quad i = 1, 2, \ldots, n \; , \tag{1}$$

where x_i are the design variables, p_i are the nonlinearity parameters and n is the number of variables. In its final form, when intervening variables y_i are substituted by original design variables x_i, the Two-Point Adaptive Nonlinearity Approximation, *TANA-3*, is defined as follows

$$\tilde{f}(\mathbf{x}) = f(\mathbf{x}^{(k)}) + \sum_{i=1}^{n} \frac{\partial f(\mathbf{x}^{(k)})}{\partial x_i} \cdot \frac{(x_i^{(k)})^{1-p_i}}{p_i} \cdot \left[x_i^{p_i} - \left(x_i^{(k)}\right)^{p_i} \right] + \frac{1}{2} \cdot \varepsilon(\mathbf{x}) \cdot \sum_{i=1}^{n} \left[x_i^{p_i} - \left(x_i^{(k)}\right)^{p_i} \right]^2, \quad (2)$$

where

$$\varepsilon(\mathbf{x}) = \frac{H}{\sum_{i=1}^{n}\left[x_i^{p_i} - \left(x_i^{(k-1)}\right)^{p_i} \right]^2 + \sum_{i=1}^{n}\left[x_i^{p_i} - \left(x_i^{(k)}\right)^{p_i} \right]^2}, \quad (3)$$

$$p_i = 1 + \frac{\ln\left[\dfrac{\partial f(\mathbf{x}^{(k-1)})}{\partial x_i} \Big/ \dfrac{\partial f(\mathbf{x}^{(k)})}{\partial x_i} \right]}{\ln \dfrac{x^{(k-1)}}{x_i^{(k)}}}, \quad (4)$$

and

$$H = 2\left\{ f(\mathbf{x}^{(k-1)}) - f(\mathbf{x}^{(k)}) - \sum_{i=1}^{n} \frac{\partial f(\mathbf{x}^{(k)})}{\partial x_i} \cdot \frac{\left(x_i^{(k)}\right)^{1-p_i}}{p_i} \cdot \left[\left(x_i^{(k-1)}\right)^{p_i} - \left(x_i^{(k)}\right)^{p_i} \right] \right\}. \quad (5)$$

In Eq. (2) $\mathbf{x}^{(k)}$ is the current design point and $\varepsilon(\mathbf{x})$ is the diagonal element of Hessian matrix. In order to simplify the calculation of $\varepsilon(\mathbf{x})$, Eq. (3), the utility constant H, Eq. (5) is introduced.
Besides its high accuracy [Xu and Grandhi 1998], the main advantage of *TANA-3* is that nonlinearity parameters p_i, as well as $\varepsilon(\mathbf{x})$ are defined in a closed form by matching the gradients and function values at previous design point. Defined in this way, the approximation parameters and the whole approximation process is computationally inexpensive.
A more complete description of the above approximation scheme is given in [Xu and Grandhi 1998].

2.2 An illustrative example
Cost function of the two-dimensional Cam design problem [Schittkowski 1987]

$$f(\mathbf{x}) = \frac{\pi}{3.6}\sum_{i=1}^{100}\left\{\left[\ln(t_i) + x_2\sin(t_i) + x_1\cos(t_i)\right]^2 + \left[\ln(t_i) + x_2\cos(t_i) - x_1\sin(t_i)\right]^2\right\}, \quad (6)$$

$$t_i = \pi\left[1/3 + (i-1)/180\right], \quad i = 1,...,100, \quad (7)$$

in the neighbourhood of the design point $\mathbf{x} = (0.46, 0.27)$, could be easily approximated using *TANA-3* ($f(\mathbf{x}) = 92.63$; $\nabla f(\mathbf{x}) = (80.83, 153.14)$; $H = 6.58$, $\varepsilon(\mathbf{x}) = 14.85$; $p_1 = 2$; $p_2 = 1.08$) as

$$\tilde{f}(\mathbf{x}) = 92.63 + 87.29\left(x_1^2 - 0.21\right) + 157.47\left(x_2^{1.08} - 0.24\right) + \\ + 7.43\left[\left(x_1^2 - 0.21\right)^2 + \left(x_2^{1.08} - 0.24\right)^2\right]. \quad (8)$$

Applying the design change vector $\mathbf{d} = (0.22, 0.19)$, the cost function at the new design point $\mathbf{x}^* = (0.68, 0.46)$, using Eq. (8) instead of Eqs. (6) and (7), is then estimated as $\tilde{f}(\mathbf{x}^*) = 145.17$, which is 1.42% lower than the actual function value of $f(\mathbf{x}^*) = 147.26$.

3. Implementation and testing

Presented function approximation method is tested against a set of 18 real-life design optimization problems [Hock and Schittkowski 1981; Schittkowski 1987], Table 1. In that table *n* denotes the number of design variables and *m* denotes the number of constraints.

For evaluation purposes *TANA-3* approximation scheme, Eqs. (2) to (5), were incorporated in the line search routine of the *RQPopt* design optimization package [Magazinović 2003]. During the test runs *TANA-3* approximations were performed alongside with the ordinary *RQPopt* routines using exact function values. Applied in that way it is straightforward to determine the error of each approximation performed. The actual number of executed approximations during the each test run is stated in the last two columns of Table 1. In total, 97 cost and constraint functions were approximated and overall 7049 function approximations were performed.

Table 1. Test problems

No.	Problem description	*n*	*m*	Number of approximations	
				Cost function	Constraint functions
1	Transformer design	6	2	29	58
2	Reactor design	8	6	9	35
3	Heat exchanger design	8	6	36	167
4	Static power scheduling	9	6	156	936
5	Chemical equilibrium	10	3	32	96
6	Alkylation process	10	11	155	1059
7	3-stage membrane separation	13	15	473	2922
8	Gear train of minimum inertia	2	0	11	–
9	Journal bearing design	2	1	4	4
10	Cam design	2	2	33	24
11	Flywheel design	3	2	8	6
12	Whirlpool design problem	3	1	2	2
13	Welded beam design	4	5	28	47
14	Coupler curve problem	6	4	37	11
15	Alkylation process	7	14	7	54
16	Multi-spindle automatic lathe design	10	15	14	69
17	Chemical equilibrium	12	3	34	49
18	Synthetic natural gas production problem	48	3	111	331

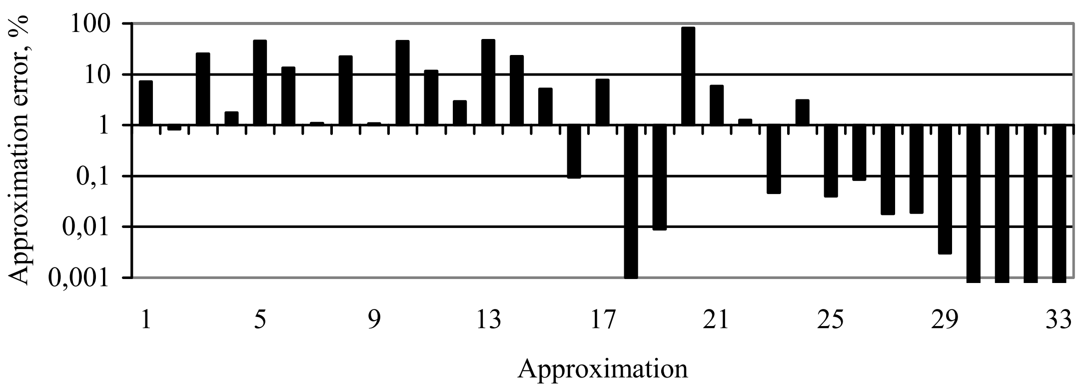

Figure 1. An example of approximation error history (Cam design cost function)

4. Results

Each approximation from the last two columns of Table 1 is classified into one of the six error classes according to the achieved error level. Results are summarized in Table 2.
The upper part of Table 2 comprises the percentage difference (relative error) between the 1179 exact and approximated cost functions. If 59 approximations with relative error greater than 25% are discarded, the mean relative error of performed approximations is 0.47%.
The lower part of Table 2 comprises the absolute difference (absolute error) between the 5870 exact and approximated constraint functions. In this case the relative difference is not suitable, since the constraint functions are usually very small, even equal to zero. If 968 approximations with the absolute error greater than 0.25 are discarded, the mean absolute error of performed approximations is 0.018.

Table 2. Numerical test results

Cost functions approximation						
Relative error class, %	≤0.1	1.0	5.0	10.0	25.0	>25.0
No. of approximations	1005	53	30	14	18	59
Constraint functions approximation						
Absolute error class	≤0.001	0.010	0.050	0.100	0.250	>0.250
No. of approximations	2758	844	774	227	299	968

5. Conclusions

The majority of approximations performed by the Two-Point Adaptive Nonlinearity Approximation algorithm possess high accuracy and low computational cost. Therefore, this method is highly appropriate for application in design and structural optimization. However, it should be clearly noted that some 15% of the performed approximations exhibit significant inaccuracy.
Beside the application in the design and structural optimization, the Two-Point Adaptive Nonlinearity Approximation could be useful whenever repetitive calculation of computationally expensive functions, even the implicit ones, are needed.

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