

A14 – A NEW MATH MODEL FOR GEOMETRIC TOLERANCES TO PROMOTE  
INTERCHANGEABILITY IN MANUFACTURING: A KEY TO SUCCESS IN  
GLOBAL AND NON-COLLOCATED DESIGN

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## Introduction

Existing methods for analysing tolerances today are based strongly on ad-hoc conventions from engineering practice and more weakly on mathematical principles. Consequently, full three-dimensional analysis of tolerances in assemblies is not done today, and contemporary design software is only partially compatible with existing standards. The authors have been developing a new mathematical model for geometric tolerances since 1995. The main objectives are to formalize tolerance specifications and enable full 3-D tolerance analysis. The examples and conclusions in this presentation are not new, but instead are drawn from our recent research publications. The objective of this paper is to acquaint a wider, and largely different, audience to the importance of this problem for designers and to recent advances in this area of research.

## Tolerances are important

When poor choices for tolerances are made, it costs money and impacts competitiveness. Specifying small tolerances so that variations from the design parameters are minimal seems like a safe haven for designers, but it drives up the cost of manufacturing for the product. Yet, when tolerances are more liberal, mistakes are made too often. As one example of an expensive retrofit for interchangeable parts, tolerances were well selected to control variations of three target features (two faces and one axis) that were intended to locate a sub-assembly on a bigger machine. But this entire system of tolerances was referenced to the wrong datum that engaged a feature on the machine—a datum that itself was permitted to have very liberal variations. The consequent degradation of performance resulted in retrofitting 10,000 machines, having six sub-assemblies each, at a cost of US\$ 6,500 per sub-assembly: nearly US\$ 400M total.

## Geometric tolerances: standards of interpretation

The modern procedures for specifying limitations to manufacturing variations are contained in the ASME/ANSI Standard Y14.5M and the ISO Standard 1101; when followed to the letter, these ensure proper communication among engineers, an aspect important in non-collocated design. The ANSI Standard classifies dimensional variations (size, position) and geometric variations (form, orientation, profile, runout) into separate classes. This is because the types of variation that need to be controlled depend on functional and assembly requirements. For example, form needs to be controlled for smooth motion, perpendicularity is important for insertion of long features, and feature

size and location must be controlled for proper assembly. Although far from a comprehensive list, the Standard also calls for

- the allowed variation for each tolerance class to be defined with a tolerance-zone in which the target entity is permitted several degrees of freedom for displacement.
- certain tolerance zones to float within other zones (e.g. the form tolerance zone has floating position and orientation within a size or orientation zone, yet an orientation zone is permitted only to translate inside a size zone).
- tolerances to be applied to both resolved entities (axes, mid-planes), and to boundary elements (faces, edges).
- the opportunity for "bonus tolerances" wherein, as just one example, position variations can be traded for size variations.
- datum precedence, i.e. multiple datums having an ordered sequence that is to be used when quality control personnel make verifying measurements on the part or assembly.

The Standards themselves are not based on any mathematical model but it contains valuable experiential knowledge collected from decades of engineering practice.

## Tolerance-analysis software and its limitations

The purpose of computer-aided tolerancing (CAT) software is to assist designers in choosing tolerances that are both small enough to ensure function, yet as large as possible to minimize cost. Any CAT software is structured from a math model of some sort, and, as described in a contemporary survey [1], each math model for existing commercial software omits one or more of the items in the above list from the standards. Further, these systems still require the designer or a specialist to be part of the decision loop, typically, for example, deciding about which features to use as datums so that variations are properly controlled. And, since tolerancing has not been treated well in the curricula of most engineering schools for many years, mistakes often are made. One example is cited above.

## The new mathematical model

This paper contains a description of a new bi-level mathematical model for geometric tolerances that has been under development at Arizona State University. In the local model, special solids are created in hypothetical Euclidean point-spaces; their dimension, size, shape, and internal subsets reflect all the variational possibilities for a target feature. Since these spaces can be created in any dimension, the model can represent bonus tolerances and floating zones. The new model is compatible with the standards for geometric tolerances.

### References

1. Shen, Z., Ameta, G., Shah, J.J., and Davidson, J.K. To appear. Details are in the paper.

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