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Contribution to an Optimized Development Process for Model Range Products Considering Uncertainties of Information

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Abstract

This paper illustrates how an elaborately organized design process in combination with a design for reliability program gains a high product reliability level. We consider this aspect for an advanced planning of a product reliability test procedure.

Our approach constitutes that the knowledge experts use while designing products is reflected in the results of a subsequent reliability test. We demonstrate that design engineers transfer knowledge from one to another product. Thus, it seems to be allowed to use reliability information of previous/similar products to define the conditions of a reliability test of a new product. Methods based on the Bayes procedure are well known in this context. In the past one big disadvantage of these procedures was the unknown transformation factor. There is no method known to determine this uncertainty factor so far.

In this paper we introduce a new approach to define the transformation factor. Our method exactly defines the factor based on reliability test results. The result of this new approach is an optimized development process regarding costs and time.

Keywords: Design for reliability, test planning, prior information, uncertainty, Bayes

1. Introduction

Shorter development times contrasted with customer demands on higher reliability and product variety require a carefully thought-out and optimized product development process. In today's short product lifecycles this also pertains to the test program [1]. The higher the reliability requirements on a product are the more extensive is the test for proving the reliability targets. The classical theory of reliability demonstration tests yields a large sample-size necessary to demonstrate the product reliability [2]. To prove a reliability of $R = 90\%$ with a confidence level of $C = 90\%$ a sample-size of 22 parts is required for example. In case a company develops many similar products the test effort to testify the reliability targets of the whole model range is extremely high [3]. From the economic point of view there is a need to decrease the amount of tested parts.

In this paper we state that the obtained reliability can be considered as a proof of the quality of the design process. The general reliability approaches do not care about the way a product is designed. There is no difference whether the product is designed by experts or even by inexperienced people. The requirements to prove the reliability stay the same. However, it is obvious that a product which is designed by experts will be more reliable. This aspect has to be considered in the planning of a reliability test procedure. We do so by transferring reliability information of one product to another product. We present an approach, based on the Bayes procedure, which considers information of previous or similar products to reduce the test effort of a reliability test procedure [4], [5].

2. Use of prior information to optimize the development process

The optimization of the development process is based on an improved design process. It is well known that the design process has an enormous impact on the reliability a product will obtain under customer use. An elaborately organized design process gains a high reliability level.

2.1 Design for Reliability in the Design Process

The design process as defined by Pahl and Beitz [5] is shown in figure 1. The whole process is divided in four stages named "Planning", "Conceptual Design", "Embodiment Design" and "Detail Design". In figure 1 an exemplary design for reliability program is adopted to the design process. The DFR program enjoins qualitative and quantitative actions the designer should perform during the design process in order to achieve a higher product reliability level. It encompasses reliability methods including FMECA, FTA, HALT / HASS or parts derating for example. If the DFR program is well elaborated and the designer uses it correctly, one important result of the DFR program will be the increase of the designer's reliability knowledge at the end of the design process.

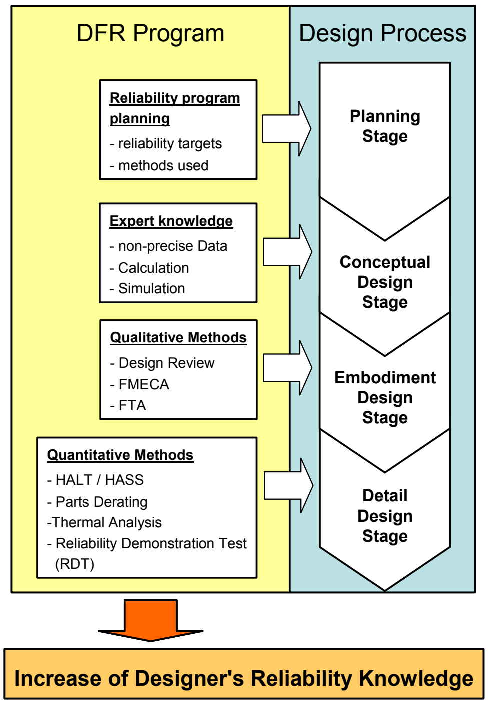

Figure 1. DFR program in the design process

Regarding model range products the context of figure 1 has an interesting effect. A Model range includes similar but not identical products and thus design process is performed for all products of the model range.

For simplification reasons the following comments refer to the exemplary model range shown in figure 2. It includes only the two products 1 and 2. First of all the designer performs the design process for product 1 as defined in figure 1. The last step of the design process is the reliability demonstration test to prove the reliability of this product. Due to the claimed reliability targets of product 1 a specific sample-size has to be tested. As mentioned above 22 parts are needed to prove a reliability of 90% with a confidence level of 90% for example. If all tested parts overcome the required lifetime, the design of product 1 is verified. Herewith the design process of product 1 is completed and the designer starts to design product 2 by means of adjustment, redesign etc. But the designer does not start from scratch. Product 2 receives an optimized design process due to the experience the designer gained while developing product 1. This context is shown in figure 2. The designer automatically transfers knowledge from product 1 to product 2. Again at the end of the design process of product 2 a test procedure has to be performed to finally prove the product reliability. Following the classical theory of test planning [2] also for product 2 a sample of 22 parts has to be tested, claiming the same reliability targets as product 1. However, this seems to be illogical. Although the design process of product 2 is optimized and the probability that product 2 will be more reliable is increased, the requirement regarding the test effort stays the same. Thus it seems to be allowed to use the prior reliability information of product 1 to define the conditions of the reliability demonstration test of product 2. This procedure will result in a reduced sample-size necessary to demonstrate the reliability targets of product 2.

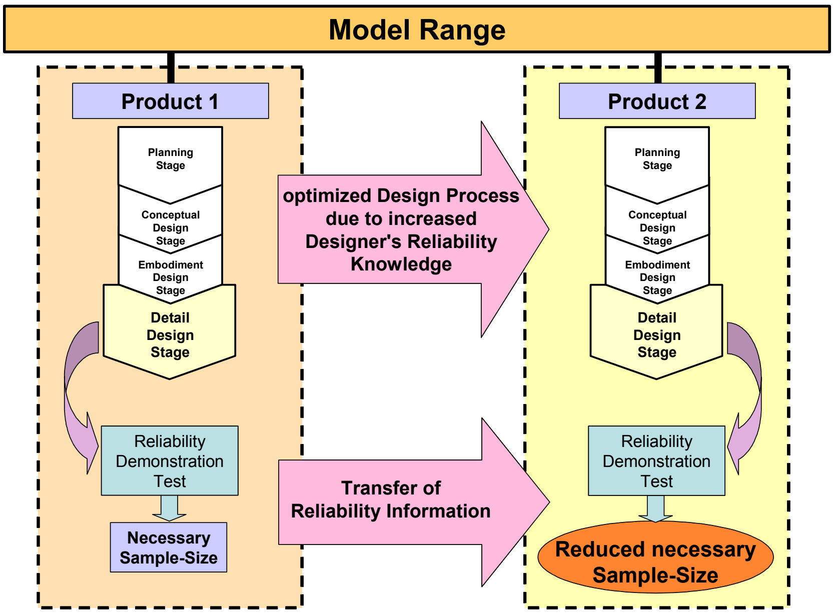

Figure 2. Transfer of information between model range products

To generalize this justification of reliability information transfer between products, one can state that a designer can be treated as an expert. Furthermore experts own much more knowledge regarding product design than inexperienced people like students for example. As mentioned above the test effort of a reliability demonstration test does not depend on the way a product is designed. Thus, it seems generally allowed to transfer reliability information whenever it is possible.

Reliability tests generate failure times of each model range product. To transfer reliability information of product 1 to the test planning of product 2 it has to be mathematically described.

3. Transfer of reliability information

The Bayes procedure [4], [5], [7] is a well known method to transfer prior reliability information. Figure 3 schematically shows the procedure for the model range of figure 2. The prior information of product 1 is known from its reliability demonstration test and can be described by a distribution. This distribution is called prior density of product 1 in figure 3. Also some parts of product 2 have been tested. This information is described by prior density 2. This amount of product 2 is not jet enough to prove its claimed reliability targets. Thus, there is a necessary, additional sample-size required. The combination of these three information sources by means of the Bayes procedure results in the so called posterior density. This density gives a better estimation of the expected reliability of product 2. The whole procedure results in a reduction of the necessary sample-size of product 2.

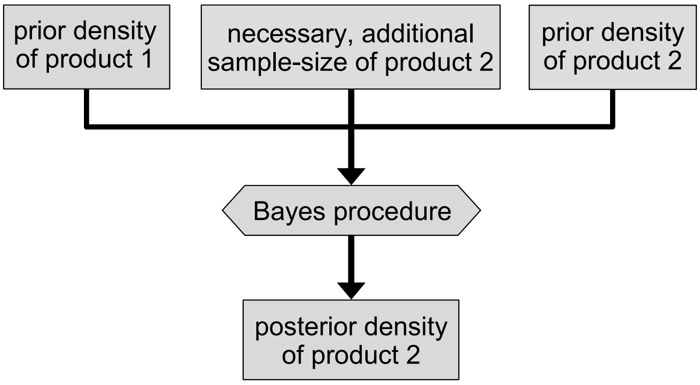

Figure 3. Combination of prior reliability information by means of Bayes procedure

One big disadvantage of the Bayes procedure is the constraint that the reliability information has to originate from a common population. Otherwise the statistical verification of the reliability may not be correct and the product will not work reliable under customer use. This constraint is often not fulfilled since prior information is known from similar products like a forerunner or products of the same model range for example.

3.1 General description of prior reliability information

In [5] a new procedure was suggested by Krolo. This procedure, in the following called Krolo-procedure, allows considering variable information sources as prior information for the planning of a reliability test. Due to the introduction of the so called transformation factor the constraint of the Bayes procedure is fulfilled. This transformation factor artificially reduces the quality level of prior reliability information with regard to the different populations of the information sources. It is obvious that the information of a similar product may not be totally transferable to the actual product. Thus, the total use of prior information is not recommended.

Test results, field data, results of fatigue damage calculations as well as exact failure times of tested parts can be considered as input information for the Krolo-procedure. The procedure generally describes prior reliability information by means of beta distributions. The advantage is that the posterior density is a beta distribution as well and thus it is easier calculated. The beta distribution is defined by its two parameters A and B. These parameters depend on the sample-size n and the rank i and are defined by eq. (1):

$$A = n - i + 1 \qquad B = i \ . \tag{1}$$

The density of the beta distribution is calculated from eq. (2) regarding the reliability R as random variable:

$$f(R) = \frac{1}{\beta(A,B)} R^{A-1}(1-R)^{B-1} \ . \tag{2}$$

In the following the mathematical description of prior information required for the case study of chapter 5 is introduced. The whole Krolo-procedure can be gleaned in [5].The parameters *A* and *B* of prior reliability information have the index 0 for clarity reasons.

Sufficient number of failures during test procedure

The Krolo-procedure describes the product failure behaviour by a two parametric Weibull distributions. This distribution is defined by the shape parameter *b* and the scale parameter *T* [2]. If sufficient failures occur during the reliability test procedure, it will be legitimate to express this information by a distribution from the statistical point of view. The parameters of the beta distribution for such a type of prior information can be calculated from eq. (3):

$$A_0 = n - (n+0.4)\left(1-\left(1-F_p(t_s)\right)^{\frac{1}{r^b}}\right)+0.7$$
$$B_0 = (n+0.4)\left(1-\left(1-F_p(t_s)\right)^{\frac{1}{r^b}}\right)+0.3\ . \qquad (3)$$

Here, *n* is the sample-size, *r* is the acceleration factor (if needed) and *b* is the shape parameter of the Weibull distribution. $F_p(\mathrm{t_s})$ defines the failure probability at the required lifetime t_s by eq. (4)

$$F_p(t_s) = 1 - \mathrm{e}^{-\left(\frac{t_s}{T}\right)^b}\ . \qquad (4)$$

With eq. (1) and (2) the parameters of the prior density are given.

Insufficient number of failures during test procedure

Otherwise, if only few failures occur during a test procedure, describing the failure behaviour by a distribution seems to be critical. To solve this statistical problem, the Krolo-procedure includes the exact failure times of tested parts in the calculations. Thus, the prior density is described by eq. (5):

$$A_0 = n - \left(\sum_k i_k{}' - 1\right)$$
$$B_0 = \left(\sum_k i_k{}' - 1\right) + 1\ , \qquad (5)$$

where *n* is the number of tested parts and $\mathrm{i_k}'$ is the rank of the accordant failure time. Each $\mathrm{i_k}'$ is defined by eq. (6):

$$i_k{}' = 1.4\left(1-0.5^{\frac{1}{\left(\frac{t_{ak}}{t_s}\right)^b r_k{}^b}}\right)+0.3\ . \qquad (6)$$

Necessary sample-size

Afterwards, the posterior distribution of the reliability is generated with the actual sample distribution by means of Bayes procedure [4], [5]. The result is the posterior distribution of product 2 which yields the necessary sample-size for demonstrating the reliability targets of this product. Since the prior density of the prior information is described by a beta distribution the posterior density is a beta distribution as well. Its parameters A and B can be calculated from eq. (7):

$$A = \sum_{i=1}^{p} \Phi_i A_{0i} + n$$
$$B = \sum_{i=1}^{p} \Phi_i \left(B_{0i} - 1\right) + 1 \ . \qquad (7)$$

Here, p is the number of the given prior densities and $A_{0\mathrm{i}}$ and $B_{0\mathrm{i}}$ are the parameters of these densities. In eq. (7) n is the sample-size needed to prove the claimed reliability targets of product 2 and Φ is the transformation factor of the corresponding prior information.

The confidence level C can be calculated by eq. (8) depending on the reliability R, the necessary sample-size n and the chosen transformation factor Φ:

$$C = \int_{R(t_s)}^{1} \frac{1}{\beta(A,B)} R^{A-1} (1-R)^{B-1} dR \ . \qquad (8)$$

The goal of the approach described in chapter 2 is to determine the necessary sample-size of product 2 in order to prove its claimed reliability targets. If the values of reliability and confidence level are known the necessary sample-size can be numerically calculated from eq. (8).

4. Determination of the transformation factor

Due to the fact that the model range products are not identical there is an uncertainty in transferring information from one to another product. There are differences concerning the geometry, the load or the material of the components for example. Thus, the total use of prior information is not recommended. Most important point is to consider and describe these uncertainties. Therefore one has to define the so called transformation factor which describes the uncertainty of prior reliability information.

In this chapter we introduce an approach to define the transformation factor. The reliability demonstration tests of the reliability level in figure 2 generate failure times of tested parts for each product. By means of these test results it is possible to define the transformation factor.

As shown in figure 4 the constraint of the Bayes procedure regarding the population of information causes a restriction of the input information. Therefore, the Krolo-procedure considers uncertainties of prior reliability information as mentioned in chapter 3. Thus, one has to define the transformation factor which describes the transfer rate of prior reliability information. The transformation factor is defined as $0 \leq \Phi \leq 1$ [5].

Reviewing the Bayes constraint the transformation factor has to analyse the origin of reliability information. It has to compare the populations of the different input information.

So called rank tests, as described in [8] and [9], determine the probability P that two samples draw from the same population. This probability correlates with the transformation factor. Thus, the transformation factor is generally defined as $\Phi = P$. Furthermore, one can state the following consequences. If common populations of prior reliability information are obtained, a transformation factor $\Phi = 1$ will be considered and the prior reliability information is completely transferred to the test planning. Otherwise, if the samples draw from slightly different populations the prior information will be only partially considered referring to the probability value of the rank test, $0 < \Phi < 1$. Totally different populations are described with

$\Phi = 0$ and it is not allowed to transfer reliability information between the products. The approach introduced in this paper uses the rank test found by Kolmogoroff and Smirnoff [8] to determine the transformation factor.

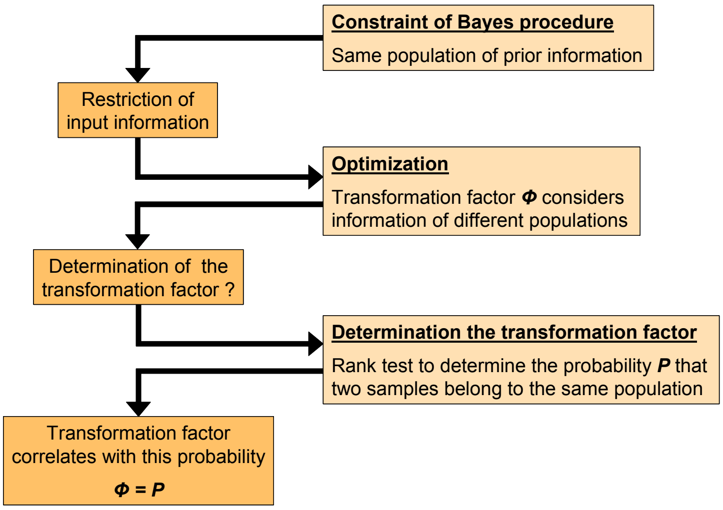

Figure 4. Approach to determinate the transformation factor

The Kolmogoroff-Smirnoff test compares the empirical cumulative distribution functions of two samples (figure 5) and is known as the sharpest homogeneity test [9]. Regarding to the statistics of Kolmogoroff and Smirnoff the transformation factor Φ is calculated as

$$\Phi = 1 - \binom{m+n}{m}^{-1} \cdot \sum_{i=0}^{h} \pi_m^{\pm}(i) \ . \tag{9}$$

Here, m and n are the sizes of the samples. By repeated use of

$$\pi_j^{\pm}(i) = \pi_{j-1}^{\pm}(i+n) + \pi_{j-1}^{\pm}(i+n-m) + \pi_{j-1}^{\pm}(i+n-2\cdot m) + \ldots + \pi_{j-1}^{\pm}(i+n-n\cdot m) \ , \tag{10}$$

$\pi_m^{\pm}(i)$ can be calculated. The initial and boundary conditions of eq. (10) are described in [8]. In eq. (9) h is the test statistic and is defined as

$$h = m \cdot n \cdot D \ , \tag{11}$$

where D is the greatest observed difference between the two empirical cumulative distribution functions of the samples as shown in figure 5.

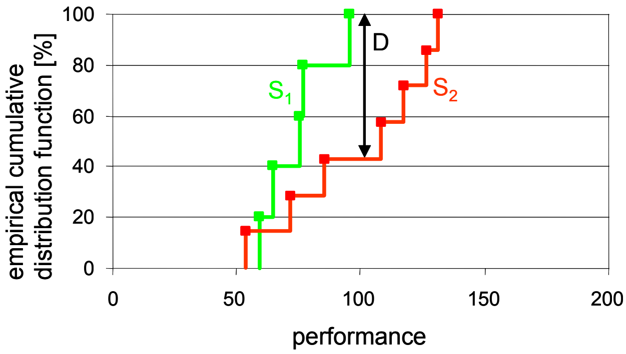

Figure 5. Test situation of the Kolmogoroff-Smirnoff rank test

Now, the transformation factor can be exactly defined. It is possible to perform a reliability test planning considering prior reliability information. Input information of similar products, of a forerunner or field data can be correctly transferred between products by means of the Krolo-procedure. The benefit of this optimized test planning is a reduction of the necessary sample-size of the verification test. Thus, the whole development process especially of model range products is optimized. Due to the use of prior information shorter development times and lower development costs are obtained.

5. Case Study

This chapter demonstrates the new approach to determine the transformation factor by means of a case study.

In [3] an improved reliability test procedure for axle gears based on the Krolo-procedure was presented. The model range of the axle gears includes three different gear ratios. Here, reliability information of two gear ratios is considered to reduce the sample-size of the verification test of the third gear ratio. In the following we refer to only two of these gear ratios, namely 43:12 and 40:13. In this case study we use the reliability of gear ratio 40:13 to reduce the test effort of gear ratio 43:12. We increase the reliability targets of 43:12 compared with the ones in [3] to clearly demonstrate the effects of the procedure. A reliability of $R = 90\%$ at a lifetime of $t_s = 0.12$ with a confidence level of $C = 90\%$ is required to verify the design of 43:12. Due to a performed test procedure some failure times of both 43:12 and 40:13 are known. These standardised failure times are shown in table 1.

Table 1. Failure times of tested parts

gear ratio	failure times (standardised)	gear ratio	failure times (standardised)
43:12	0.33 0.45 0.87	40:13	0.42 0.61 0.70 0.74 1.20 1.21 1.27

First of all the prior reliability shown in table 1 has to be mathematically described as mentioned in chapter 3. In the following prior information of gear ratio 40:13 has the index 1 and information referring to 43:12 has the index 2 for clarity reasons.

The failure data of seven parts is available for gear ratio 40:13. This information is expressed by a Weibull distribution, figure 6. The result is the description of its failure behaviour by a two parametric Weibull distribution. The scale parameter T is found with $T = 1$ and the shape parameter b with $b = 2.48$. Since failure modes are identical for all gear ratios of the model range this shape parameter will be used as information for transmission ratio 43:12 [2]. With eq. (3) and (4) the parameters of the prior density of 40:13 are given as

$$A_1 = 7.53 \qquad \text{and} \qquad B_1 = 0.47 \,.$$

The occurred failures of gear ratio 43:12 also have to be described by a prior density. As a result, this information can be used to reduce the sample size of 43:12 that has to be tested additionally. Since only three failure times are known for 43:12 describing its failure behaviour by a distribution seems to be critical. Therefore, we consider the exact failure times to calculate the prior density of this gear ratio. The parameters are found by eq. (5) and (6) to

$$A_2 = 2.98 \qquad \text{and} \qquad B_2 = 1.02 \,.$$

For further calculations the transformation factor Φ_2 for the prior information of 43:12 was logically set to $\Phi_2 = 1$ because the parts already tested and the parts that still have to be tested are identical.

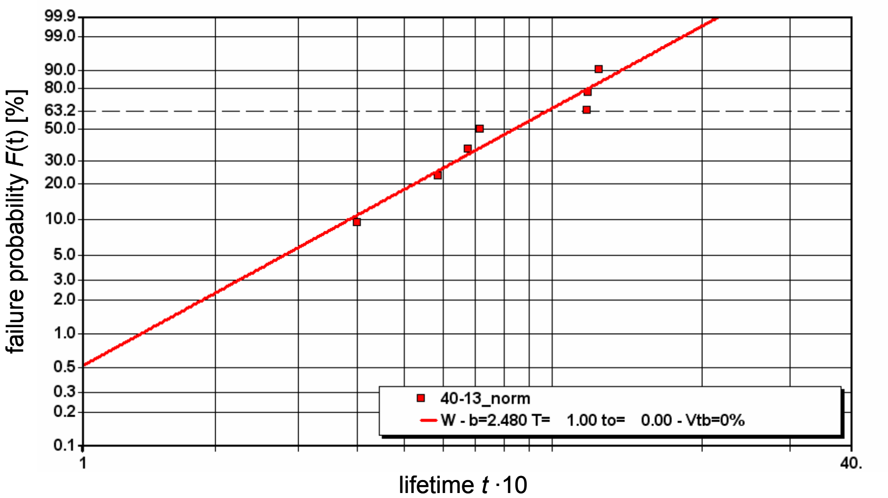

Figure 6. Weibull distribution of gear ratio 40:13

Hence, the prior information is described, the additional sample-size necessary to prove the design of gear ratio 43:12 can be calculated by eq. (7) and (8).

Results

Due to the reliability requirements of gear ratio 43:12 the classical theory [2] yields a necessary sample-size of

$$n = \frac{\ln(1-C)}{\ln R} = \frac{\ln(1-0.9)}{\ln 0.9} = 22 \,. \tag{11}$$

Eq. (11) is known as the so called Success Run-Test. Based on this test planning 19 parts of 43:12 are additionally needed to obtain the reliability targets.

Figure 7 shows the necessary sample-size achieved under consideration of the prior information of 40:13. The result of the analysis varies depending on the transformation factor Φ_1. As one can see it is generally possible to reduce the additional sample-size for 43:12 if results of 40:13 are considered. If no prior information is taken into account ($\Phi_1 = 0$), 19 parts will be needed. This correlates with the Success Run-Test of eq. (11). By changing the transformation factor up to the maximum of $\Phi_1 = 1$ a theoretical reduction of 16 parts is acquired.

However, it was not possible to exactly determine the additional test effort so far. Due to the fact the transformation factor could not be defined correctly, it was only possible to demonstrate the theoretically achieved reduction as shown in figure 7.

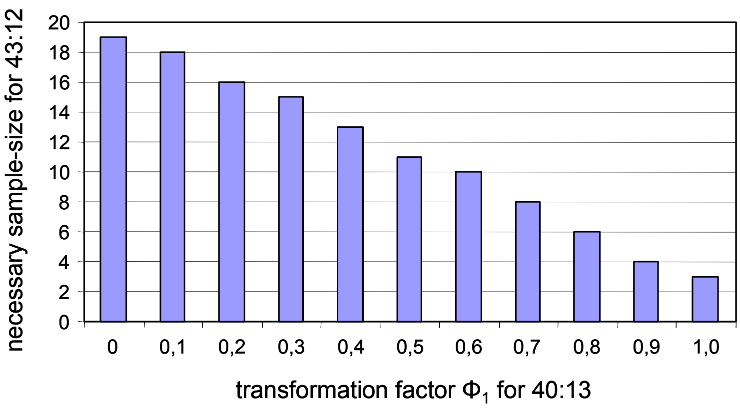

Figure 7. Necessary (additional) sample-size depending on the transformation factor Φ_1

With the approach to determine the transformation factor as described in chapter 4 it is possible to solve this problem. In figure 8 the empirical cumulative distribution functions of gear ratios 43:12 and 40:13 are shown. The greatest difference between this two functions is found with $D = 0.524$.

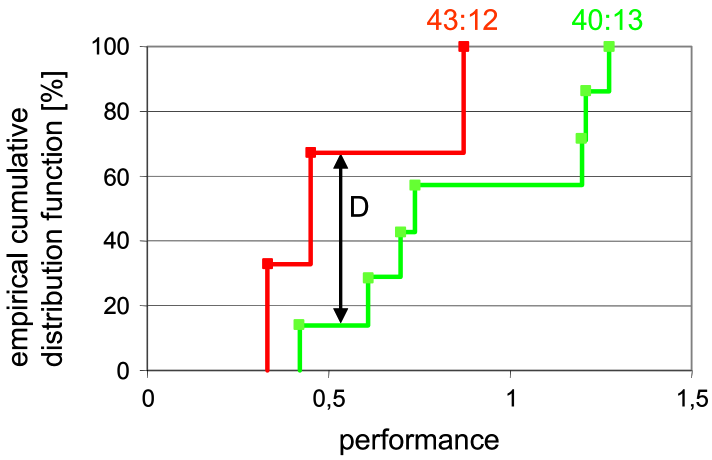

Figure 8. empirical cumulative distribution functions of 43:12 and 40:13

The test statistic h is calculated from eq. (11) to

$$h = m \cdot n \cdot D = 3 \cdot 7 \cdot 0{,}5238 = 11 \ ,$$

where m and n are the numbers of tested parts.

By means of the Kolmogoroff -Smirnoff rank test the transformation factor can be found by eq. (9) with

$$\Phi = 1 - \binom{m+n}{m}^{-1} \cdot \sum_{i=0}^{h} \pi_m^{\pm}(i) = 0.4 \ .$$

Thus, the correct transformation rate is $\Phi_1 = 0.4$ to consider reliability information of gear ratio 40:13 for the test planning of gear ratio 43:12.

With refer to figure 8 the additional sample-size needed to prove the reliability of 43:12 is exactly 13 parts. Compared with the classical test planning a reduction of 6 parts is achieved. If a test procedure is performed with these 13 parts and no failures occurred before the required lifetime of $t = 0.12$, the over-all test effort of gear ratio 2 will be 16 parts

(13 additionally tested and 3 already tested). This represents a well optimized development process regarding costs and time, not only for gear ratio 43:12 but also for the whole model range.

6. CONCLUSION

In this paper we illustrated how an elaborately organized design process in combination with a design for reliability program gains a high reliability level. From this we stated that the obtained reliability can be considered as a proof of the quality of the design process. The general approaches to verify product reliability do not care about the way a product is designed. There is no difference whether the product is designed by experts or even by inexperienced people like e.g. students. The requirements to prove the reliability stay the same. However, it is obvious that a product which is designed by experts will be more reliable. Our approach constitutes that the knowledge experts used while designing products is reflected in the results of the subsequent reliability test. The better the design process is the more reliable the product will be.

By means of model range products we demonstrated that design engineers already applied their experience and knowledge on other products of the model range. Engineers again use this knowledge to design neighbourhood products. We illustrated this transfer of knowledge from one to another product. Thus, we stated that it seems to be allowed to use reliability information of previous/similar products to define the conditions of a reliability demonstration test of a new product.

We presented an approach, based on the Bayes procedure, which considers prior reliability information. By means of the so called Krolo-Procedure a reduction of the test effort of a test procedure is achieved. One disadvantage, not only of this method but also of all methods based on the Bayes procedure, was the unknown transformation factor. There was no method known to determine this factor so far. In the past it was chosen to the best of one's knowledge.

In this paper we introduce a new approach to define this uncertainty factor. Based on the rank test of Kolmogoroff and Smirnoff we showed how the transformation factor can be exactly calculated by means of test results.

We applied our approach on a case study and defined the additional sample-size necessary to obtain the reliability requirements of a model range product. By means of the new reliability test procedure it was possible to prove the reliability of model range products with less tests than the classical theory enjoins. The result was an optimized development process regarding costs and time.

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