INTERNATIONAL CONFERENCE ON ENGINEERING DESIGN
ICED 05 MELBOURNE, AUGUST 15 - 18, 2005

APPROACH TO A SYSTEMATIC METHODOLOGY FOR UNIVERSAL DESIGN – CONSIDERATION OF QUANTITATIVE USER DIVERSITY USING INTERVAL ARITHMETIC

Tamotsu Murakami and Masatoshi Deguchi

Abstract

Recently, the concept of "universal design", which is to design products equally usable by diverse users as much as possible, has become important. To enable designers to more certainly and efficiently design products which offer maximum usability for diverse users and are internationally marketable, a general and systematic methodology for universal design is important. As a methodology of supporting systematic idea generation for universal design, in this paper, we propose a structure-control approach matrix which should systematically cover all possible design approaches of product structure and control for user diversity. Also, in this paper, we propose a methodology of representing quantitative user diversity as intervals and conducting their calculations using interval arithmetic to evaluate, compare and optimize design ideas considering quantitative user diversity in a general and systematic manner. The proposed methodology and implemented software are applied to two simplified design problems considering quantitative user diversity: a design of a button height on a wall and a design of three dimensional layouts of cellular phone buttons. Through calculation results and a simple user test, we confirmed that the proposed methodology and implemented software can yield adequate information and hints for universal design.

Keywords: User diversity, usability, structure-control approach matrix, interval arithmetic, inclusive design.

1. Introduction

Recently, the concept of "universal design" [1], which is to design products equally usable by diverse users as much as possible, regardless of age, gender, physical height/strengths, disability or nationality, has become important [2]. We have been studying the general and systematic design methodology of generating design ideas to deal with user diversity and of evaluating, comparing and optimizing the ideas for all intended diverse users. Such methodology should enable designers to more certainly and efficiently design products which offer maximum usability for diverse users and are internationally marketable. As two examples of such design methodology, in this paper, we propose a structure-control approach matrix to support systematic idea generation and design calculation using interval arithmetic to consider quantitative user diversity.

2. Structure-control approach matrix for universal design

Although a single product equally usable by all diverse users is the ideal goal of universal design, such an approach is not always possible, practical or effective. Therefore, we should consider that the goal of universal design is to provide all diverse users with easily usable

Table 1. Design approaches of product structure to support user diversity

Approach				Description
Single structure	Fixed structure	Same usage	Common	Uses some common property independent of user diversity (e.g., pictogram is commonly understandable regardless of language differences).
			Mitigated	Mitigates usable condition for disadvantaged users without sacrificing usability for advantaged users (e.g., large character displays are good for users with good eyesight as well as those with poor eyesight).
			Average	Designs structure considered to be most convenient for the average user while at the same time acceptably convenient for nonaverage users.
			Disadvantageous-oriented	Designs structure considered to be more convenient for disadvantaged users than for advantaged users (e.g., operation buttons situated at low position for wheelchair users can also be used by walking users).
		Juxtaposition of different uses		Juxtaposes different uses in single structure (e.g., operation buttons with both visual and braille notations).
	Variable structure			Uses variable structure to cover user diversity (e.g., height-adjustable chair).
Basic structure + optional structure				Composes basic structure, which alone is complete product, and optional user-diversity-dependent structure (e.g., bicycle and training wheels).
Common structure + individual structure				Composes user-diversity-independent structure and user-diversity-dependent structure (e.g., frame of glasses is user-eyesight-independent whereas lenses are user-eyesight-dependent).
Individual structures				Provide individual structures for user diversity (e.g., shoes of different sizes).

Table 2. Structure-control approach matrix for universal design

Structure / Control	Single structure						Basic structure + optional structure	Common structure + individual structure	Individual structures
	Fixed structure					Variable structure			
	Same usage				Juxtaposition of different uses				
Control	Common	Mitigated	Average	Disadvantageous-oriented					
No control (Manual)	- Pictogram	- Tableware with easy grip - Stationary usable with less force	- (many)	- Operation buttons situated only at low position for wheelchair users	- Buttons with both visual and braille notations - Juxtaposition of long and short straps in trains - Bifocal glasses	- Chair of adjust-able height	- Bicycle + training wheels	- Glasses (frame + lens)	- Shoes (of different sizes)
Single control / Constant control	- Automatic door						- Automo-bile + optional lift for wheel-chair		
Single control / Variable control / Manually switched		- Radio for the elderly (with adjustable speed and tone)			- Elevator with different actions (e.g., wait time for door to close) according to buttons (high- or low-positioned) - Cellular phone using bone conduction	- Electric reclining sheet		- Hearing aid	
Single control / Variable control / Automatically switched	- Electric tool with auto-matic learning function					- Electric assistant bicycle			
Basic + optional controls	- Software product + language localiza-tion patch								
Individual controls									- Individual-ly ordered product

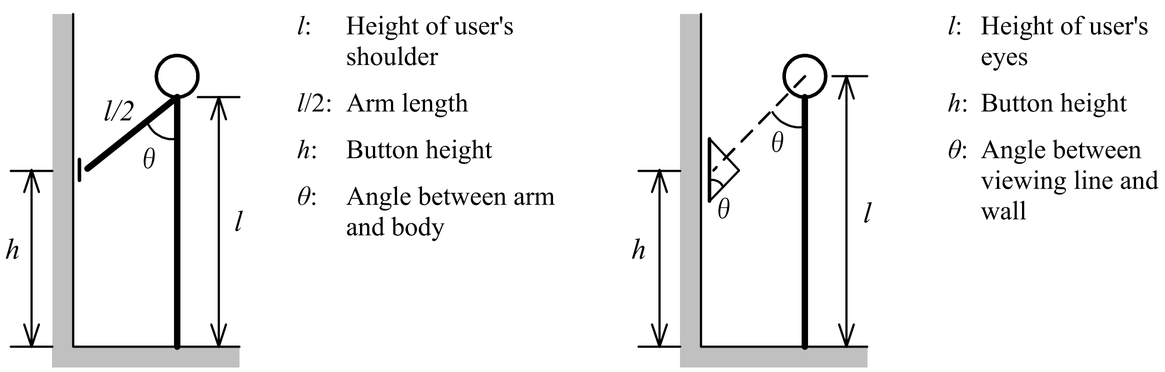
products which can be either common ones to cover all users or different ones to cover every user group depending on the product characteristics. Before the concept of universal design was proposed, engineers had already been designing many products considering their use by diverse users. Through a deductive thought of possible design approach enumeration and an inductive analysis of existing product examples, we classified the possible design approaches of product structure to deal with diverse users as listed in Table 1. Similarly, we also classified possible design approaches of product control (including software and power assist). Since an engineering product in general consists of structures and controls, a matrix of such structure and control design approaches should systematically cover all possible approaches to universal design (Table 2). This matrix can support designers in the following manners.

- This matrix should help a designer to investigate every possible design idea systematically and avoid missing good design ideas.
- By collecting design examples for usability and classifying them in this matrix, as shown in Table 2, a designer can create a universal design database (e.g., as web pages).

3. Universal design methodology using interval arithmetic

3.1 Design evaluation, comparison and optimization for diverse users

Once designers have come up with a number of ideas using the structure-control approach matrix, then they need feasibility estimation, comparison and selection, as well as optimization of the ideas for not just specific (e.g., average or typical) users but all intended diverse users. The authors classify user diversity in general into two types: qualitative diversity such as difference of languages or availability/complete-unavailability of vision, and quantitative diversity such as height and physical strength. The following method in this paper focuses on quantitative diversity.



(a) Pushability of button (b) Visibility of button

Figure 1. Design problem of button height on a wall

Figure 1 is a simple example of designing button height on a wall. The relationship between variables in Figure 1(a) is expressed by equation (1), which is transformed to equation (2).

$$h = l \cdot (1 - \frac{1}{2}\cos\theta) \tag{1}$$

$$\theta = \arccos\left(2(1 - \frac{h}{l})\right) \tag{2}$$

Here, we introduce a simple quantitative definition of button pushability p (ease of pushing the button).* We assume if we can push a button without raising an arm at all (θ=0) then it is the easiest case (p=1), and if we need to fully raise an arm ($\theta = \pi$) to push a button then it is the hardest case (p=0). By linearly interpolating the easiest and hardest cases, pushability p is defined by equation (3), which is transformed to equation (4).

$$p = 1 - \frac{\theta}{\pi} \tag{3}$$

$$\theta = (1 - p)\pi \tag{4}$$

By substituting equation (4) for equations (1) and (2), we obtain equation (5) to calculate button height h to provide users of height l with pushability p, and equation (6) to calculate the pushability p for button height h for users of height l.

$$h = l \cdot \left(1 - \frac{1}{2}\cos\left((1 - p)\pi\right)\right) \tag{5}$$

$$p = 1 - \frac{1}{\pi}\arccos\left(2(1 - \frac{h}{l})\right) \tag{6}$$

Similarly we introduce a simple quantitative definition of button visibility v (ease of reading the button). Since the area of a button on a wall to be seen is determined by $\sin\theta$ as in Figure 1(b), we define visibility by equation (7), which is transformed to equation (8).

$$v = \sin\theta \tag{7}$$

$$\theta = \arcsin v \tag{8}$$

Although l and θ in Figure 1(a) and those in Figure 1(b) are not exactly the same, we approximate them as equal to simplify the problem. By substituting equation (8) for equation (1), we obtain equation (9) to calculate button height h to provide users of height l with visibility v. By transforming equation (9), we obtain equation (10) to calculate the visibility v for button height h for users of height l.

$$h = l \cdot \left(1 - \frac{1}{2}\cos(\arcsin v)\right) \tag{9}$$

$$v = \sin\left(\arccos\left(2(1 - \frac{h}{l})\right)\right) \tag{10}$$

Here, we define the usability u of a button by combining pushability and visibility using equation (11). If we assume only users of a specific height l (e.g., 1.5m), we can easily analyze the relationship between button height h and its usability u for the intended users using equation (11) as in Figure 2 and can obtain the optimal design solution h to provide maximum usability as approximately 1.2m to 1.3m.

$$u = \frac{p + v}{2} = \frac{1}{2}\left\{1 - \frac{1}{\pi}\arccos\left(2(1 - \frac{h}{l})\right) + \sin\left(\arccos\left(2(1 - \frac{h}{l})\right)\right)\right\} \tag{11}$$

* Note that the design problems and some indices (e.g., pushability, visibility, usability and ease of finger posture) are described or introduced in this paper only to explain our proposed methods and calculation frameworks. Actual design problems and indices such as usability should be more complicated [3].

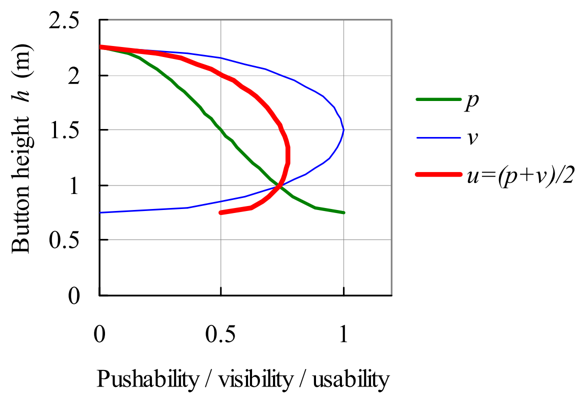


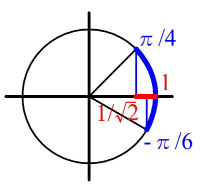
Figure 2. Relationship between button height and its usability for specific users (l=1.5)

If we assume users of specific size or dimensions, now a software tool to simulate and evaluate usability of a product is available [4][5]. In universal design, however, we need to consider users of quantitative diversity and to obtain (in some sense) an "optimal" solution for all intended users. For example, if we assume users from children to tall adults then user height l may range from 1.2m to 1.8m and we need to determine "optimal" button height for all of them.

3.2 Quantitative user diversity consideration using interval arithmetic

To support such consideration of quantitative user diversity systematically, we propose a methodology and computational framework based on interval arithmetic [6]. An interval is a range of numbers bounded by its lower and upper ends, and interval arithmetic is a system for performing arithmetic not on numbers but on intervals. When a variable x may take a value ranging from its minimum value x_L and maximum value x_U ($x_L <= x <= x_U$), we represent the closed interval as $X = [x_L, x_U]$ using a capital-letter variable according to conventions in interval arithmetic. Similarly interval arithmetic function names are represented by capital letters. Equation (12) shows some examples of interval arithmetic operators. As equations (12) and Figure 3 indicate, interval arithmetic operators and functions yield possible value ranges. Therefore, defining the quantitative diversities of users as intervals and performing interval-arithmetic-based design calculations should be a general and systematic methodology and tool for estimation, comparison and optimization of design ideas for universal design.

$$
\begin{aligned}
&X = [x_L, x_U], \quad Y = [y_L, y_U] \\
&X + Y = [x_L + y_L, x_U + y_U] \qquad X - Y = [x_L - y_U, x_U - y_L] \\
&X * Y = [\min(x_L * y_L, x_L * y_U, x_U * y_L, x_U * y_U), \max(x_L * y_L, x_L * y_U, x_U * y_L, x_U * y_U)] \\
&X / Y = [\min(x_L / y_L, x_L / y_U, x_U / y_L, x_U / y_U), \max(x_L / y_L, x_L / y_U, x_U / y_L, x_U / y_U)]
\end{aligned} \tag{12}
$$



$$\mathrm{COS}([-\frac{\pi}{6}, \frac{\pi}{4}]) = [\frac{1}{\sqrt{2}}, 1] \tag{13}$$

Figure 3. Example of interval arithmetic function

Some studies have been reported, which use intervals to represent the possibility and uncertainty of parameter values particularly at the early stage of design [7][8][9][10]. In this research, we use intervals to represent quantitative user diversity in a general and systematic manner.

3.3 Implementation of interval arithmetic calculation framework

Although some interval arithmetic programming libraries are available [11], we implemented our own interval arithmetic library using C++ and OpenGL graphics because of possible flexibility and extendibility to combine with some other features as graphics or to introduce design-support oriented functionality. Presently we implemented the interval arithmetic version of C++ operators and the functions '+', '-', '*', '/', 'sin', 'cos', 'tan', 'asin', 'acos', 'atan', 'sqr', 'sqrt' and 'pow', which are necessary to handle the design problems described in this paper.

The merit of interval arithmetic is that interval arithmetic calculation yields an interval which includes the correct answer. Interval arithmetic, however, may yield unnecessarily wide intervals (called "excess intervals") as its result as follows, where the correct answer should be $Y = [-0.5, 0.5]$.

$$\begin{aligned} &X = [0, 2\pi] \\ &Y = \mathrm{SIN}(X) * \mathrm{COS}(X) = [-1, 1] * [-1, 1] = [-1, 1] \end{aligned} \tag{14}$$

One way of avoiding this problem is to transform an expression so that one variable appears only once in the expression and then to conduct interval arithmetic as follows.

$$\begin{aligned} \sin\theta * \cos\theta &= \frac{1}{2}\sin 2\theta \\ Y &= \frac{1}{2}\mathrm{SIN}(2 * X) = [-0.5, 0.5] \end{aligned} \tag{15}$$

Another way of narrowing resulting excess intervals is to conduct interval arithmetic for divided intervals and then to merge the resulting intervals as follows.

$$\begin{aligned} &X = [x_1, x_n] = [x_1, x_2] \cup [x_2, x_3] \cup \cdots \cup [x_{n-1}, x_n] \quad (X_i = [x_i, x_{i+1}]) \\ &\bigcup_{i=1}^{n-1} F(X_i) \subseteq F(X) \quad (F : \text{interval arithmetic function}) \end{aligned} \tag{16}$$

Currently we use these two methods to narrow excess intervals. The second method is implemented as a program which divides an interval X into two subintervals, conducts interval arithmetic calculation for each of them and merges the resulting intervals recursively. The recursive interval division ends either when another depth of division does not narrow excess interval over a prespecified ratio or when the depth reaches the prespecified depth limit.

4. Design problem 1: button height on wall

Although interval arithmetic is a general calculation method that can be used for any type of design problem, this paper focuses on the design problems of shape and space to provide usability for diverse users.

4.1 One-button approach

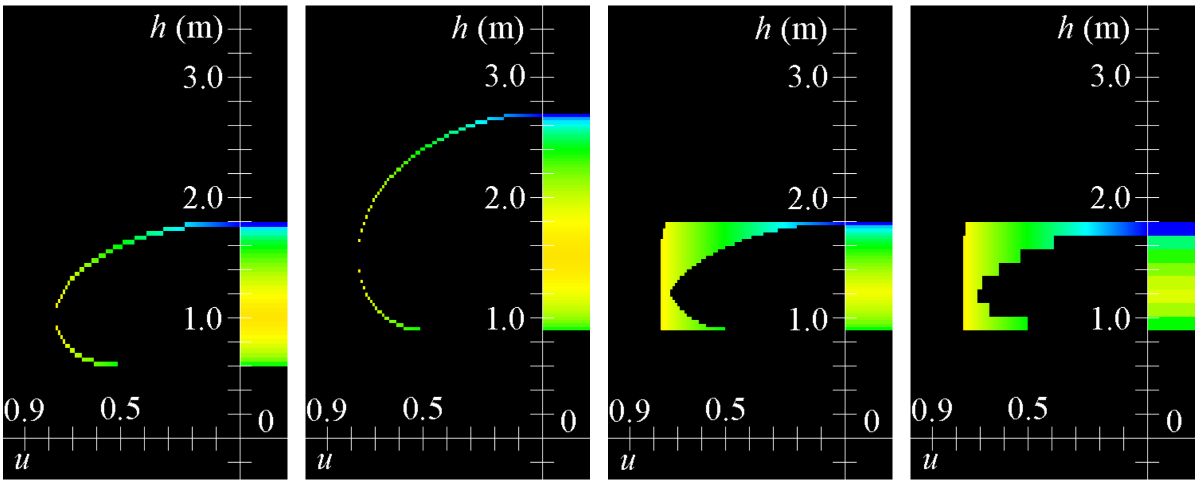
As a first design problem example, we consider designing a button height on a wall (described in section 3.1) using interval arithmetic. This can be regarded as a one dimensional space layout problem. First, we prepare an equation (17) which is an interval arithmetic version of equation (11). We apply dividing intervals and merge the results for interval L (user height), as described in section 3.3, to reduce excess interval.

$$U = \frac{P+V}{2} = \frac{1}{2}\bigcup_{\text{divide } L}\left\{1-\frac{1}{\pi}\mathrm{ARCCOS}\left(2(1-\frac{H}{L})\right)+\mathrm{SIN}\left(\mathrm{ARCCOS}\left(2(1-\frac{H}{L})\right)\right)\right\} \tag{17}$$

Equation (17) yields usability $U=[0.507, 0.773]$ for button height $H=[1.6, 1.6]$ and user height $L=[1.2, 1.8]$. This means that when the intended users' height l ranges from 1.2m to 1.8m, the usability of a button height h =1.6m may range from 0.507 for the most disadvantaged users to 0.773 for the most advantaged users. Here, we divide the user accessible height range H into 32 sections H_i $(i=1,\cdots,32)$, calculate the usability U_i of H_i for L using equation (17) and depict the spatial distribution of usability graphically as in FEM analysis.

Figure 4(a) and Figure 4(b) show the results for specific users of l=1.2 and l=1.8, respectively. In these figures, the relationship between usability and button height are depicted as lines without width as in Figure 2 because no user diversity is considered. The right-hand side of the vertical axis is painted with colors corresponding to the lower end value of the usability interval at each height. Apparently the optimal button heights for l=1.2 and l=1.8 are different.

Color scale for usability value.



(a) $L=[1.2, 1.2]$ (b) $L=[1.8, 1.8]$ (c) $L=[1.2, 1.8]$ (d) Rough division

Figure 4. Analysis of spatial usability distribution using interval arithmetic (one-dimensional)

On the other hand, Figure 4(c) depicts the result for diverse users $L=[1.2, 1.8]$. In this case, the usability at each button height has some range which appears as horizontal width in the figure. From Figure 4(c) the following information is suggested.

- There are both advantaged users and disadvantaged users for every button height.
- The maximum usability of every height is similarly high, probably because every height

has its advantaged users (a high button is convenient for tall users and a low button is convenient for short users).

- The minimum usability of every height differs.

If we support all intended users with a single button (as in "single structure (same usage)" approach in Table 2), one possible strategy of optimizing button height is to maximize the minimum usability. According to this strategy, we suggest that the button height should be approximately 1.2m where the minimum usability is maximum as shown in Figure 4(c), which means the button ensures at least a usability u=0.75 for all intended users.

By sampling l=1.2, 1.3, ..., 1.8 and h=0.9, 1.0, ..., 1.8 and conducting arithmetic (not interval arithmetic) calculation for their every combination, spatial distribution of usability can also be depicted. In that case, however, the usability between calculated spatial points is unknown. On the other hand, interval arithmetic ensures that the correct answer is included in the obtained interval and never missed. That may enable us to first analyze the usability distribution roughly with wide interval sections as in Figure 4(d), where the button height range is divided into eight sections, and then to analyze only the promise range with fine interval sections as in Figure 4(c) where the button height range is divided into 32 sections.

4.2 Two-button approach vs. one-button approach

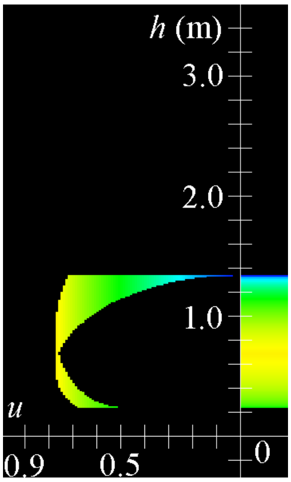
Next, we consider supporting more user diversity by including people in wheelchairs. We model people in wheelchairs by modifying the user height in Figure 1 as 0.75m<=l<=1.2m ($L_W = [0.75, 1.2]$) and arm length as 4/5l. By modifying equation (17) we obtain equation (18).

$$U_W = \frac{P+V}{2} = \frac{1}{2} \bigcup_{\text{divide } L_W} \left\{ 1 - \frac{1}{\pi}\text{ARCCOS}\left(5(1-\frac{H}{4L_W})\right) + \text{SIN}\left(\text{ARCCOS}\left(5(1-\frac{H}{4L_W})\right)\right) \right\} \quad (18)$$

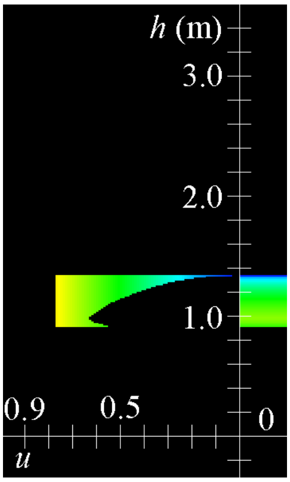
We divide user accessible height range H into 32 sections H_i $(i = 1, \cdots, 32)$, calculate usability U_i of H_i for L_W using equation (18) and depict the spatial usability distribution graphically as in Figure 5(a). This result suggests that a button height of approximately 0.7m should ensure at least a usability of approximately 0.77 for all intended user heights $L_W = [0.75, 1.2]$. Therefore, if we prepare both a high-positioned button on the basis of data shown in Figure 4(c) and a low-positioned button on the basis of data shown in Figure 5(a) at the same time (as in "single structure (juxtaposition of different uses)" approach in Table 2), we can provide a usability of approximately 0.75 to 0.77 for all intended user diversities $L \cup L_W = [0.75, 1.2] \cup [1.2, 1.8] = [0.75, 1.8]$ with two buttons.

If we try to support all intended user diversity $L \cup L_W = [0.75, 1.8]$ with a single button as described in section 4.1, the possible button height range is an intersection of those shown in Figure 4(c) and in Figure 5(a), and the usability range is a union of those shown in Figure 4(c) and in Figure 5(a). As a result, we obtain the usability distribution as shown in Figure 5(b). This figure suggests that this single button provides some of the intended users with a usability of approximately 0.63, which is worse than that of the two-button approach.

As demonstrated here, our proposed method enables us to analyze, estimate, compare and optimize design solutions considering quantitative user diversity in a general and systematic manner.



(a) A button for only $L = [0.75, 1.2]$



(b) A button for both $L = [0.75, 1.2]$ and $L = [1.2, 1.8]$

Figure 5. Usability of button for intended users including people in wheelchairs

5. Design problem 2: cellular phone button layout

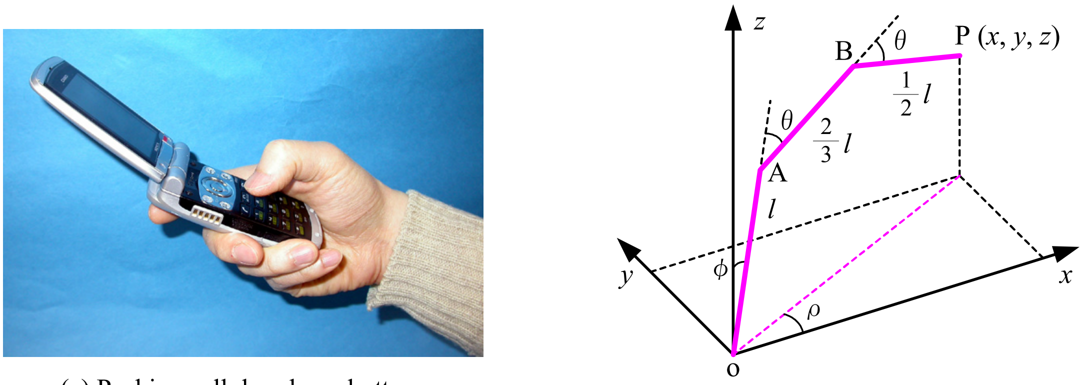
As a second design problem example, we consider designing the three-dimensional layout of cellular phone buttons using interval arithmetic. Here, we select a cellular phone as an example for the following reasons:

- Cellular phones are presently important and will prospectively become more so in the future.
- Cellular phones are used by quantitatively diverse users from children to elderly people and by people of different hand sizes.
- Designing an easily usable cellular phone, that can be held in one hand and easily operated with the thumb, should be important from the viewpoint of "ubiquitousness", i.e., anywhere, anytime.

5.1 Design problem definition

Although the design of the three-dimensional layout of cellular phone buttons is very complicated, here we simplify the problem as follows.

- Only twelve buttons ('0' - '9', '*' and '#') arranged in a rectangular area are considered.
- Buttons are pushed by the tip of the thumb (Figure 6(a)), and the thumb is modeled on three bones, three joints, one length parameter l and three angle parameters θ, ϕ, ρ (Figure 6(b)).
- We try to achieve a button layout in which the thumb tip can reach every button with ease without assuming a hard posture.



(a) Pushing cellular phone buttons with thumb fingertip

(b) Simplified thumb model

Figure 6. Simplified model of cellular phone button layout problem

Here, we quantitatively define the ease of thumb posture when the fingertip P is at a position (x, y, z). We assume the bend angles at joints A and B are the same as θ to remove redundant degrees of freedom. By the derivation described in Appendix, we obtain equations (19), (20) and (21) to calculate the joint angles θ, ϕ and ρ from the fingertip position x, y, z.

$$\theta = \arccos\left\{-\frac{1}{2}+\frac{1}{12l}\sqrt{-14l^2+72(x^2+y^2+z^2)}\right\} \tag{19}$$

$$\begin{aligned}\phi = {} & \frac{\pi}{2}-\arccos\left\{-\frac{1}{2}+\frac{1}{12l}\sqrt{-14l^2+72(x^2+y^2+z^2)}\right\}-\arccos\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \\ & +\arcsin\left[\frac{1}{2\sqrt{x^2+y^2+z^2}}\sin\left\{\arccos\left(-\frac{1}{2}+\frac{1}{12l}\sqrt{-14l^2+72(x^2+y^2+z^2)}\right)\right\}\right]\end{aligned} \tag{20}$$

$$\rho = \arctan(y/x) \tag{21}$$

Here, we define the ease of a joint to take a maximum value of 1 when the joint bends at a relaxation angle and takes a minimum value of 0 when the joint bends to the upper limit or bends (stretches) to a lower limit angle (Figure 7). By assuming the ease between lower limit, relaxation and upper limit angles is linearly interpolated and assuming the lower limit angle is 0, the upper limit angle is $\pi/2$, the relaxation angle is $\pi/4$ for the thumb joint angles θ, ϕ and ρ, ease of joint for the three angles e_θ, e_ϕ and e_ρ are defined by equation (22). Here, we define the ease of thumb posture e as the sum (mean) of ease of three joints (four angles) as equation (23).

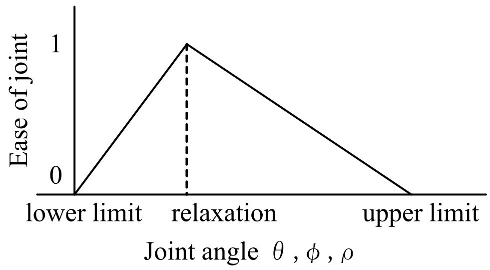


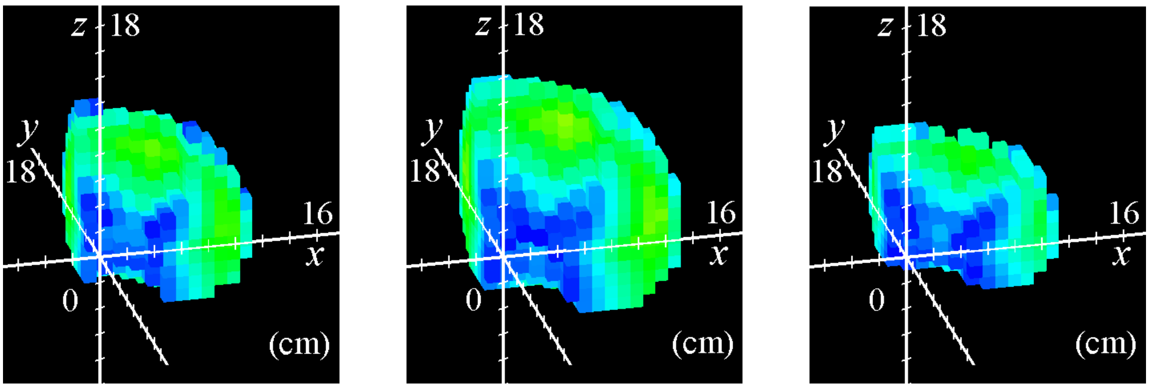
Figure 7. Relationship between ease and angle of joint

$$e_\theta = 1 - \left|1 - \frac{4\theta}{\pi}\right| \qquad e_\phi = 1 - \left|1 - \frac{4\phi}{\pi}\right| \qquad e_\rho = 1 - \left|1 - \frac{4\rho}{\pi}\right| \tag{22}$$

$$e = (e_\theta + e_\theta + e_\phi + e_\rho)/4 \tag{23}$$

5.2 Analysis of usability distribution in three dimensional space

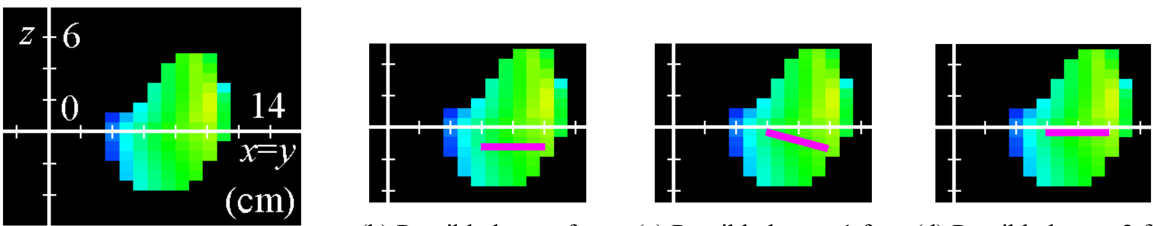
We regard the usability of a button at a position (x , y , z) in three-dimensional space as the ease of thumb when the fingertip is at the position. We divide the three-dimensional space ranges X , Y , Z into sections X_i , Y_j , Z_k and conduct calculation to obtain the usability interval E of spatial sections X_i, Y_j, Z_k for users of thumb length L (Figure 6(b)) using interval arithmetic version of equations (19) - (23). Figure 8 shows the result in which each spatial section cube is painted with a color of the minimum value of usability E calculated for the section in the color scale shown in Figure 4. Figure 8(a) and Figure 8(b) show usability distribution difference by thumb length difference, and Figure 8(c) shows the usability distribution for all intended diverse users in the same as in Figure 4(c).



(a) For $L = [5.5, 5.5]$ (cm) (b) For $L = [6.5, 6.5]$ (cm) (c) For $L = [5.5, 6.5]$ (cm)

Figure 8. Three-dimensional spatial distribution of usability

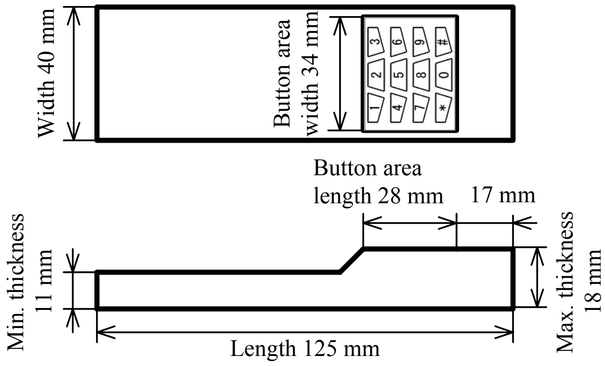
From Figure 9(a), which shows a section of Figure 8(c) by the $x = y$ plane, we can see that the low position near the thumb root provides poor usability. If we assume only the three-dimensional regions below z =0 in Figure 9(a) are available for the button layout design because a cellular phone must be within some thinness, such button layouts as in Figure 9(c)(d) should be better than those shown in Figure 9(b) because they avoid poor usability regions.



(a) Section by $x = y$ plane (b) Possible layout for poor usability (c) Possible layout 1 for better usability (d) Possible layout 2 for better usability

Figure 9. Usability distribution for section by $x = y$ plane ($L = [5.5, 6.5]$ (cm))

To confirm calculated usability difference by different designs, we conducted a user test for 30 students (undergraduate third year in mechanical engineering) who do not know about this research at all. We made six paper mock-ups of designs A - F as shown in Figure 10 and Table 3 and asked the students to try the mock-ups and select one or two in which they feel they can easily reach all the buttons with their thumb. We also asked the students to measure their thumb lengths in a manner in Figure 10(b).

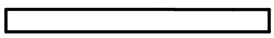
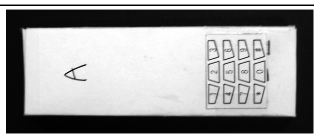
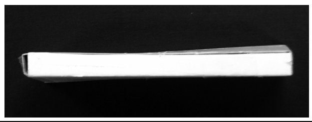
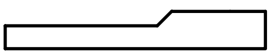
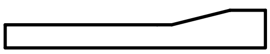
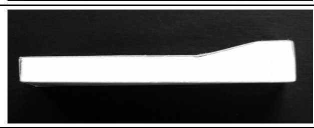
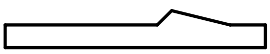
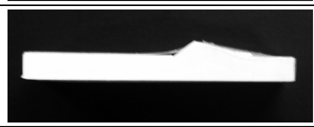
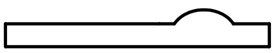
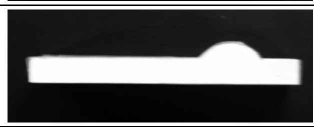
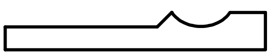
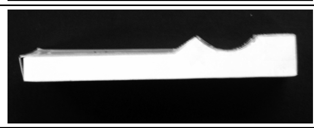


(a) Common dimensions for six designs

(b) Thumb length measurement
(Note that thumb length here is different from l in Figure 6(b).)

Figure 10. User test method

Table 3. User test result

Design name	Design shape	Mock-up photograph	Votes	Voters' thumb lengths (mm)
A		 	7	50, 50, 53, 60, 65, 65, 70
B			12	50, 56, 57, 60, 60, 60, 60, 60, 62, 63, 70, 70
C			15	50, 50, 50, 50, 55, 60, 63, 64, 65, 65, 70, 70, 70, 70
D			8	50, 55, 58, 60, 60, 60, 60, 65
E			0	
F			1	56
		Total	43	

As Table 3 shows, mock-up C (which corresponds to Figure 9(c)) and mock-up B (Figure 9(d)) got more votes than mock-up A (Figure 9(b)). This result matches the estimation in Figure 9. Our simplified analysis in this paper, however, does not apparently include some issues to explain the poor test result for mock-up F whose layout also avoids the poor usability region in Figure 9. Figure 11 shows relationship between preferred mock-ups and voters' thumb lengths. This is an interesting result at a glance because mock-ups B and D are preferred by users with medium thumb lengths whereas mock-up C is preferred by users with diverse (or long and short) thumb lengths. Again, our simplified analysis in this paper cannot explain the difference and further investigation is necessary.

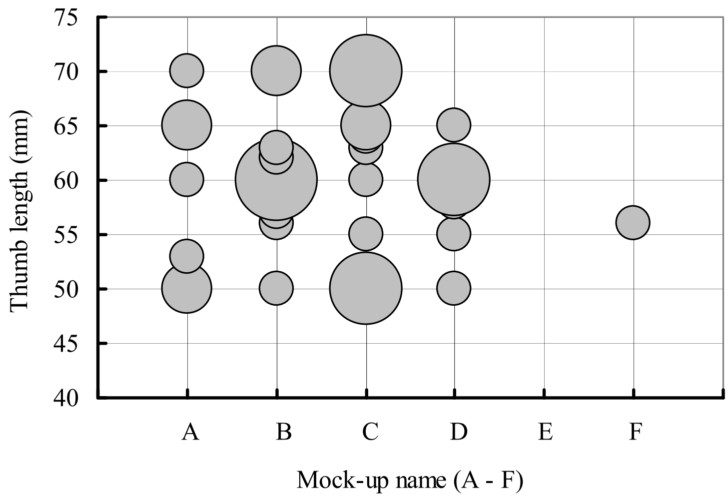


Figure 11. Relationship between preferred mock-ups and voters' thumb lengths (The circle area represents number of voters.)

6. Conclusions

(1) As a systematic methodology for universal design, we proposed a structure-control approach matrix for idea generation and interval-arithmetic-based design calculation for evaluation, comparison and optimization of design ideas considering quantitative user diversity.

(2) The interval arithmetic programming library to construct software system to aid universal design process considering user diversity is implemented using C++ and OpenGL graphics.

(3) Proposed methodology and implemented software are applied to two simplified design problems considering quantitative user diversity: designing a button height on a wall (one-dimensional layout problem) and designing three-dimensional layout of cellular phone buttons.

(4) Through calculation results and a simple user test, we confirmed that the proposed methodology and implemented software can yield adequate information and help in universal design.

Our future directions include applying the interval-arithmetic-based design calculation to more detailed and precise design problems and extending the methodology for universal design to cover qualitative user diversity.

This study has been partly supported by a research grant from the Japan Institute of Construction Engineering.

References

[1] The Center for Universal Design, North Carolina State University (http://www.design.ncsu.edu/cud/).

[2] Dong H., Keates S. and Clarkson P.J., "Designers and Manufacturers' Perspectives on Inclusive/Universal Design", Proceedings of ICED '03 (CD-ROM), 2003.

[3] Gupta S.P., Keates S. and Clarkson P.J., "Usability Considerations in the Design of Handheld Electronic Devices", Proceedings of ICED '03 (CD-ROM), 2003.

[4] Marshall R., Case K., Gyi D., Oliver R. and Porter M., "Supporting 'Design for All' through an Integrated Computer-Aided Ergonomics Tool", Proceedings of ICED '01 (Design Methods for Performance and Sustainability), 2001, pp.331–337.

[5] Williams M.A. and Medland A.J., "The Creation of Techniques for the Design of Machines Compatible with Human Posture", Proceedings of ICED '01 (Design Methods for Performance and Sustainability), 2001, pp.347–354.

[6] Hickey T., Ju Q. and van Emden M.H., "Interval Arithmetic: From Principles to Implementation", Journal of the ACM, Vol.48, No.5, 2001, pp.1038–1068.

[7] Lin B.T. and Ward A.C., "Introduction to Design Inferences over Sets of Intervals", ASME Design Theory and Methodology -DTM'92-, Scottsdale, 1992, pp.187–192.

[8] Reddy R.P. and Mistree F., "Modeling Uncertainty in Selection Using Exact Interval Arithmetic", ASME Design Theory and Methodology -DTM'92-, Scottsdale, 1992, pp.193–201.

[9] Sanjay S.E. and Rinderle J.R., "Interval Propagation: Theory and Methodology", ASME Design Theory and Methodology -DTM'92-, Scottsdale, 1992, pp.203–210.

[10] Bains N. and Ward A.C., "Labeled Interval Operations Involving Non-monotonic Equations", ASME Design Theory and Methodology -DTM'93-, Albuquerque, 1993, pp.45–50.

[11] von Gudenberg, J.W., "OOP and Interval Arithmetic - Language Support and Libraries", Lecture Notes in Computer Science, Springer-Verlag, Vol.2991, 2004, pp. 1–14.

For more information please contact:
Tamotsu Murakami
The University of Tokyo, Department of Engineering Synthesis
Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan
Tel: +81-3-5841-6327 Fax: +81-3-3818-0835
E-mail: murakami@design.t.u-tokyo.ac.jp
URL: http://www.design.t.u-tokyo.ac.jp/

Appendix: derivation of thumb model in section 5.1

We consider isosceles trapezoids OABC on a thumb plane as in Figure A-1.

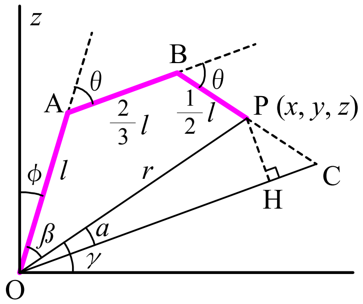


Figure A-1. Thumb plane

Here, the lengths of OH and HP are represented by equations (A-1), (A-2) and (A-3). By solving equation (A-3) for θ, we obtain equation (19).

$$\mathrm{OH} = l \cdot \cos\theta + \frac{2}{3}l + \frac{1}{2}l \cdot \cos\theta = \frac{3}{2}l \cdot (\frac{4}{9} + \cos\theta) \tag{A-1}$$

$$\mathrm{HP} = \frac{1}{2}l \cdot \sin\theta \tag{A-2}$$

$$\mathrm{OH}^2 + \mathrm{HP}^2 = \{\frac{3}{2}l \cdot (\frac{4}{9} + \cos\theta)\}^2 + (\frac{1}{2}l \cdot \sin\theta)^2 = r^2 = x^2 + y^2 + z^2 \tag{A-3}$$

In Figure A-1, the relationships between angles given by equations (A-4) and (A-5) stand. By substituting equation (A-5) for equation (A-4), we obtain equation (A-6). Also, angles α and β are represented by equations (A-7) and (A-8). By substituting equations (19), (A-7) and (A-8) for equation (A-6), we obtain equation (20).

$$\phi = \frac{\pi}{2} - \beta - \gamma \tag{A-4}$$

$$\theta = \alpha + \beta \tag{A-5}$$

$$\phi = \frac{\pi}{2} - \theta - \gamma + \alpha \tag{A-6}$$

$$\alpha = \arcsin\left(\frac{l\sin\theta}{2\sqrt{x^2 + y^2 + z^2}}\right) \tag{A-7}$$

$$\gamma = \arccos\left(\frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}\right) \tag{A-8}$$