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# THE IDENTITY OF FREEFORM SHAPE FEATURES

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## Abstract

The identification of freeform shape features is crucial in shape analysis and design knowledge retrieval. This paper focuses on the exploration of the geometric invariance of a freeform shape feature, and the identity measurements are derived based on the invariant attributes of the Fourier spectrum, including the Fourier shape identity, the normalized Fourier shape identity, the surface curvature-based intrinsic shape identity, and the cumulative shape identity. Effectiveness of these identity definitions is analysed, and efficiencies are evaluated. Working exclusively on the spatial distribution of the freeform feature, the proposed definitions are neutral and independent of low-level geometric representations.

*Keywords: Shape description, freeform feature, Fourier transform, shape identity.*

# 1 Introduction

Shape identity is a unique description of the geometry, which is essential in shape registration. It plays an important role in invariant shape description/representation, pattern recognition, and design knowledge reuse [8] [19]. A freeform shape feature (FSF) is a high-level geometric entity representing the continuous spatial distribution without prismatic geometric components, such as sharp edges or slots, and other associate attributes. The freeform shape descriptor is of great importance in CAD model representation and identification. Conventionally, shape descriptors are categorized as (a), boundary-based descriptors; and (b), regional-based descriptors [20]. The former represents shapes according to the boundary information, such as radius, contour, and chord length; the latter represents shape by regional information, such as shape matrices based on the relative areas of the shape contained in concentric rings located in the shape centroid. In boundary-based approaches, the global and local shape matching metrics for 2D shape description include global statistical approaches based on a method of moments [3], Fourier shape descriptors (FSD) [2] [15] [18] [21], wavelet shape descriptors that take the local geometric attributes into consideration, such as curvature and slope, and multi-resolution shape descriptors [6] [10] [12]. In recent years, the study of 3D shape analysis and processing using signal processing approaches has attracted more attention [11] [15] [20] [17]. In this field, metrics and some 3D shape descriptors have been widely used in conducting spatial shape analysis [16]; for instance, methods for processing point-sampled objects using spectral-based approaches have been investigated [13]. However, most of the existing shape identity measurements are either computationally expensive [20] or lack generality [8].

Fourier transforms (FTs) have been applied in different subjects helping to explore the nature of a specific phenomenon. It seems promising to derive a unique and invariant identity for local shape representation based on its corresponding FT spectrum. This paper will concentrate on the identity definitions for FSFs using their geometric distributions, and derives the identity from the aggregations of the invariant shape attributes, namely its FT spectrum. Working exclusively with the local shape distribution, the proposed approaches are independent of low-level representations, such as data structure and topology, and capable of handling mesh models, point cloud models and NURBS-based surface models.

# 2 Fourier descriptors for 2D shapes

In pattern recognition, the FTs show their particular ability to decompose a complex shape into understandable mathematical representations, like a graph, power spectrum, or phase-based description [10]. The FSD, extended from the theory of FT, provides a unique representation of a spatial shape in the frequency domain.

## 2.1 The representation of a 2D shape

**Definition 1:** A mapping $f : S \to \mathbb{R}^2$ is called a representation of a 2D shape $S$ under $f$, where $\mathbb{R}^2$ is the 2D real space.

Obviously, $f$ should possess the following properties:

- Uniqueness: i.e., for a $s \in S$, where $s$ is an element of $S$, there is one and only one $r \in \mathbb{R}^2$, so that $f(s) = r$. This is of crucial importance for the identification of different objects without ambiguity.
- Invariance: This means that the representation of $S$ should be invariant under affine transformations, such as translation, rotation, scaling and reflecting.
- Completeness: This refers to the range of a representation, in which both global shape and local detail should be contained.
- Sensitivity: i.e., the ability of a representation to reflect easily the differences between two objects.
- Robustness: This refers to the ability of the representation under noise affection. For instance, the differentiable function $f$ behaves robust under input noise, since $dy = f(x+dx) - f(x) \approx f'(x)dx$.

A 2D shape can be represented in terms of its discrete coordinates, i.e., $s(k) = [x(k), y(k)]$, for $k = 0,1,2,\cdots,K-1$. Further, it can be treated as a complex number $s(k) = x(k) + jy(k)$, for $k = 0,1,2,\cdots,K-1$. Although the interpretation of the sequence was recast, the nature of the boundary shape itself remains unchanged [10].

## 2.2 Fourier descriptor for 2D shapes and its invariance

**Definition 2**. Let $s(k)$ for $k = 0,1,\cdots,K-1$ be a 2D discrete set $S$, then the forward discrete Fourier transform (DFT) of $s(k)$ is defined as:

$$a(u) = \frac{1}{K}\sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}, \quad u = 0,1,\cdots,K-1 \tag{1}$$

where $j = \sqrt{-1}$. The complex coefficients $a(u)$ are also called the Fourier descriptor (FD) of the 2D shape $s(k)$. And the inverse DFT of these components restores $s(k)$, i.e.,

$$s(k) = \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}, \quad k = 0,1,\cdots,K-1 \tag{2}$$

The Fourier spectrum, phase angle and the power spectrum is given by

$$P(u) = |a(u)|^2 = R^2(u) + I^2(u) \tag{3}$$

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right] \tag{4}$$

where $R(u)$ and $I(u)$ are the real and imaginary parts of $a(u)$; $|a(u)|$ is the Fourier spectrum, $P(u)$ the power spectrum and $\phi(u)$ the phase angle.

The properties of the FD are summarized in Table 1.

Table 1. Basic properties of the Fourier shape descriptor

| Transformation | Boundary | Fourier Descriptor |
|---|---|---|
| Identity | $s(k)$ | $a(u)$ |
| Rotation | $s_r(k) = s(k)e^{j\theta}$ | $a_r(u) = a(u)e^{j\theta}$ |
| Translation | $s_t(k) = s(k) + \Delta_{xy}$ | $a_t(u) = a(u) + \Delta_{xy}\delta(u)$ |
| Scaling | $s_s(k) = \alpha s(k)$ | $a_s(u) = \alpha a(u)$ |
| Starting point | $s_p(k) = s(k - k_0)$ | $a_p(u) = a(u)e^{-j2\pi k_0 u/K}$ |

The fundamental characteristics of the FSD are clearly demonstrated by the 1D DFT. However, as a matter of fact, these characteristics are directly inherited in the 2D DFT.

# 3 The Fourier model of a 3D shape

To explore the invariance of the 3D shape applying FSD a prerequisite is to find a proper representation of the shape. Even though some representations proposed to describe a 3D surface using additional variables [20], we adopt the neutral representation of the shape as the input of the FSD, from which the invariant identity will be derived.

## 3.1 The discrete representation of a 3D shape

A 3D FSF or the region of interest (ROI) on an existing model can be represented by a series of discrete sections, which represents a spatial shape distribution (a vector field) of a point set. They constitute the overall shape geometry in the form of a matrix of the sampling points.

**Definition 3.** (Sampling scheme) For a freeform shape feature with feature surface $s = s(u,v)$, $S = \{s(u_m, v_n) \mid m = 0,1,\cdots,M-1; n = 0,1,\cdots,N-1\}$ is called a spatial shape distribution, and $[0, M-1]\times[0, N-1]$ the sampling grid. When $u_k = U^k$, $S^k = \{s(U^k, v_j) \mid j = 0,1,\cdots,N-1\}$, $k = 0,1,\cdots,M-1$,

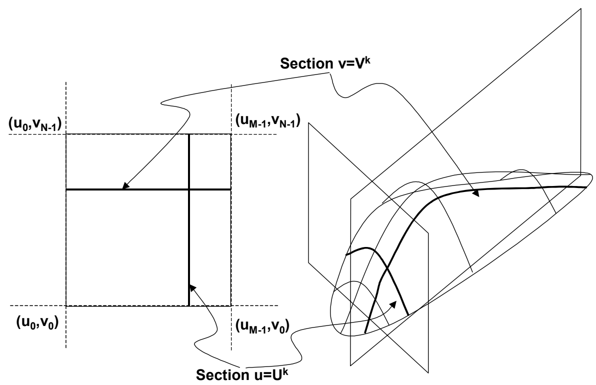


Figure 1. Sampling scheme of the freeform shape feature

is called the $k$ -th $u$ section of $S$ . The same definition can be given to $v_k = V^k$ .

The sampling distribution of a shape feature is invariant of sampling direction and starting point. For example, changing the starting point of sampling results in a variation of the row or column in the sampling matrix. But this does not change the computational property of the matrix, i.e., by transferring the row or the column the matrix could be modified without hurting its nature. Figure 1 depicts the sampling scheme, in which the correspondence between parametric domain and the spatial domain is shown. Be noticed that the sections on the sampled shape may be curved in some cases. In addition, Figure 1 is only for explanation purpose; the parametric domain is not vital. For instance, for a point cloud model, we can still obtain the sampling matrix even though it is not yet parameterized.

## 3.2 The 3D Fourier shape model

**Definition 4.** (3D Fourier shape model) A sampled distribution given by Definition (3) can be equivalently represented by means of the DFT as:

$$S = \Im^{-1}\left(F^s\right) = \Im^{-1}\left\{f^s(\tau,\zeta) | \tau \in 0,1,\cdots,M-1, \zeta \in 0,1,\cdots,N-1\right\} \quad (5)$$

where $\Im^{-1}(\cdot)$ denotes the reverse FT, $F^s$ the matrix of forward FT of $S$ , and $f^s(\tau,\zeta)$ its elements given by

$$f^s(\tau,\zeta) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} s(u,v)e^{-j2\pi\left(u\tau/M + v\zeta/N\right)} \quad (6)$$

And the Power spectrum is defined as:

$$P(\tau,\zeta) = \left\{\left(R^s(\tau,\zeta)\right)^2 + \left(I^s(\tau,\zeta)\right)^2 | \tau \in 0,1,\cdots,M-1, \zeta \in 0,1,\cdots,N-1\right\} \quad (7)$$

where $R^s(\tau,\zeta)$ and $I^s(\tau,\zeta)$ are the real and imaginary part of $f^s(\tau,\zeta)$ respectively, i.e., $f^s(\tau,\zeta) = R^s(\tau,\zeta) + jI^s(\tau,\zeta)$ and $j = \sqrt{-1}$ .

Due to the fact that the classical FT theory could be extended to arbitrary 3D grid structures with minor pre-processing [11] [17], therefore, in Equation (6), the shape distribution can be any sampled point sets, for instance, a point cloud, or a vertex set of a mesh. The 3D Fourier shape model (FSM) helps to implements the transition of the shape from spatial into frequency domain.

# 4 Identity of freeform shape features

As a matter of fact, some of the existing shape descriptors tend to be complex in representation [3] [4] [6] [15], while others are computationally expensive [16] [1] [9] [22]. Shape descriptors based on the neutral representation could help in getting rid of the complexity of the underlying model, while providing acceptable efficiency for the computation of the shape identity derived.

## 4.1 The Fourier shape identity

From the basic properties of the FT, we learn that the coefficient magnitude $\left|f^s(\tau,\zeta)\right|, \tau = 0,\cdots,M-1; \zeta = 0,\cdots,N-1$ is rotation invariant; and the magnitude of the coefficients excluding position information $\left|f^s(\tau,\zeta)\right| f^s(0,0) = 0; \tau = 1,\cdots,M-1; \zeta = 1,\cdots,N-1$ is translation invariant as well. For instance, let us suppose a rotation around $z$ -axis is being applied to Equation (6), then, the new FT of the shape distribution becomes

$$f_\theta^s(\tau,\zeta)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left(s(u,v)(\cos\theta+j\sin\theta)\right)e^{-j2\pi\left(\frac{u\tau}{M}+\frac{v\zeta}{N}\right)}=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left(s(u,v)e^{j\theta}\right)e^{-j2\pi\left(\frac{u\tau}{M}+\frac{v\zeta}{N}\right)}=f^s(\tau,\zeta)e^{j\theta},$$

where $\theta$ is the rotation angle. The magnitude of the FT spectrum element is

$$\left|f_\theta^s(\tau,\zeta)\right|=\left|f^s(\tau,\zeta)e^{j\theta}\right|=\left|f^s(\tau,\zeta)\right|\left|e^{j\theta}\right|=\left|f^s(\tau,\zeta)\right|\sqrt{\cos^2\theta+\sin^2\theta}=\left|f^s(\tau,\zeta)\right|,$$

remaining unchanged. Similar proof can be given to the rotation around a spatial vector.

**Definition 5.** (Fourier shape identity) Under the sampling scheme given by Definition 3, the FT of a FSF in Equation (6) excluding the position information, i.e., $f^s(0,0)=0$, is called the Fourier shape identity (FSI).

Evidently, the invariant properties of the magnitude of the elements of $F^s$ under affine transformation make it suitable for being an identity. However, using the $F^s$ to represent the shape feature $S$ is rather complicated, even not applicable in terms of shape knowledge indexing or reusing. For example, for a $M\times N$ matrix of the sampling points, the FSI needs to record three $M\times N$ matrixes in terms of the coordinate components. Another concern is that, the shape scaling operation should not result in a quantity change of the shape identity. This is crucial when the shape itself rather than its size is the main concern. Obviously, the FSI does not satisfy this requirement. Hence, a simplified definition is needed.

**Definition 6.** (Normalized Fourier shape identity) A normalized cumulative function defined on the Fourier shape identity with the following form

$$A^p(r)=\frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}A(r,\tau,\zeta)\left|f^s(\tau,\zeta)\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f^s(\tau,\zeta)\right|},r\in 0,1,\cdots,\frac{M}{2};M\le N \tag{8}$$

where $A(r,\tau,\zeta)=\begin{cases}1 & r-0.5\le\sqrt{\left(\tau-\frac{M}{2}\right)^2+\left(\zeta-\frac{N}{2}\right)^2}\le r+0.5\\ 0 & otherwise\end{cases}$, is called the normalized Fourier shape identity (NFSI).

NFSI $A^p(r)$ is strictly invariant of rotation, translation and scaling, e.g., for a rotation $e^{j\theta}$,

$$A_\theta^p(r)=\frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}A(r,\tau,\zeta)\left|f_\theta^s(\tau,\zeta)\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f_\theta^s(\tau,\zeta)\right|}=\frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}A(r,\tau,\zeta)\left|f^s(\tau,\zeta)e^{j\theta}\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f^s(\tau,\zeta)e^{j\theta}\right|}=\frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}A(r,\tau,\zeta)\left|f^s(\tau,\zeta)\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f^s(\tau,\zeta)\right|}=A^p(r).$$

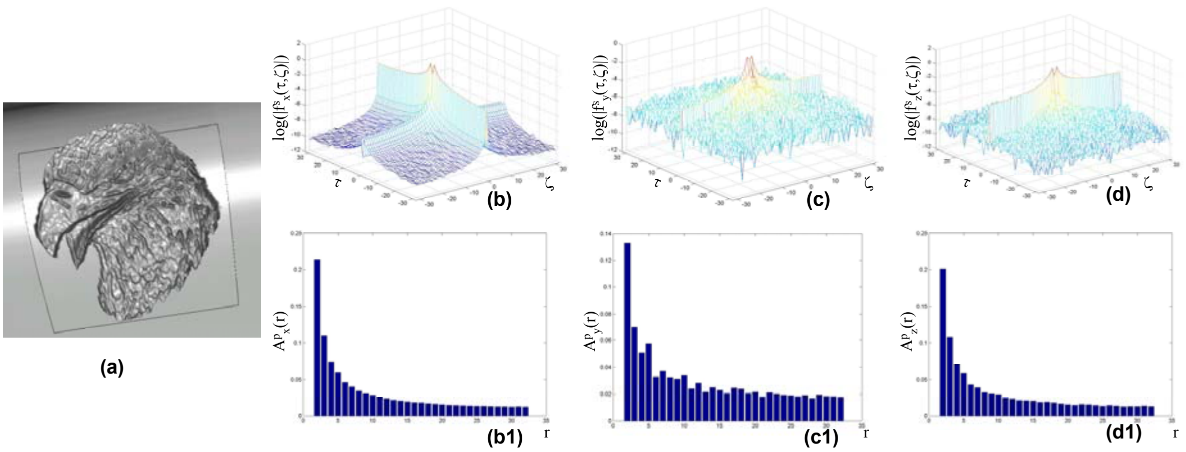


Figure 2. The Fourier shape identity of a freeform shape and its normalized Fourier shape identity. (a), The shape feature in a ROI; and its Fourier identity and corresponding normalized Fourier identity in (b), (b1)—x-coordinate component, (c), (c1) y-coordinate component, and (d), (d1) z-coordinate component.

Since the zero-frequency item is excluded from $F^s$, i.e., $\left|f^s(0,0)\right| = 0$, therefore, $A^p(r)$ is invariant of translation as well. In addition, a scaling of the shape may result

$$A_\alpha^p(r) = \frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1} A(r,\tau,\zeta)\left|f_\alpha^s(\tau,\zeta)\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f_\alpha^s(\tau,\zeta)\right|} = \frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1} A(r,\tau,\zeta)\left|\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\alpha s(u,v)e^{-j2\pi\left(u\tau/M+v\zeta/N\right)}\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\alpha s(u,v)e^{-j2\pi\left(u\tau/M+v\zeta/N\right)}\right|} = \frac{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1} A(r,\tau,\zeta)\left|f^s(\tau,\zeta)\right|}{\sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1}\left|f^s(\tau,\zeta)\right|} = A^p(r).$$

Figure 2 shows the $A^p(r)$ of a given shape feature in the ROI in terms of its $x$, $y$ and $z$-coordinate components. The Fourier spectrum in Figure 2-(b), (c), and (d) are pre-processed with logarithm, otherwise their value is too small.

## 4.2 The intrinsic shape identity

Recalling the fundamental issues in computational geometry, the curvature distribution on a surface curve is invariant of affine transformation. In fact, by considering each sampling section in Definition 3, we can immediately infer that the curvature distribution on the section contour is affine invariant, i.e., $c_i(U^k, v_n), i = 1,2; k = 0,1,\cdots,M-1; n = 0,1,\cdots,N-1$ is invariant, where $i = 1,2$ refers to the maximum and minimum curvatures. The collection of the curvatures on all sections constitutes the invariant matrix $C_i = \{c_i(u_m, v_n) \mid i = 1,2; u_m = 0,1,\cdots,M-1; v_n = 0,1,\cdots,N-1\}$, from which the intrinsic shape identity could be derived.

**Definition 7.** (Intrinsic shape identity) The distributions of the principal curvatures of a shape under the sampling scheme given by Definition 3, denoted as $C_i = \{c(u_m, v_n) \mid i = 1,2; m = 0,1,\cdots,M-1; n = 0,1,\cdots,N-1\}$, are invariant under affine transformations. Their corresponding Fourier spectrums

$$F_i^c = \left\{\left|f_i^c(\tau,\zeta)\right| \mid i = 1,2; \tau = 0,1,\cdots,M-1; \zeta = 0,1,\cdots,N-1\right\} \tag{9}$$

are called the intrinsic shape identities (ISI), which are uniquely defined by the curvature distributions, where $f_i^c(\tau,\zeta), i = 1,2$ are the components of the FT of both maximum and minimum principal curvature distributions. The distribution of the maximum principal curvature is called the first intrinsic identity $F_1^c = \left\{\left|f_1^c(\tau,\zeta)\right| \mid \tau = 0,1,\cdots,M-1; \zeta = 0,1,\cdots,N-1\right\}$; and the distribution of the minimum principal curvature is called the second intrinsic identity, denoted as $F_2^c = \left\{\left|f_2^c(\tau,\zeta)\right| \mid \tau = 0,1,\cdots,M-1; \zeta = 0,1,\cdots,N-1\right\}$.

The computation of $f_i^c(\tau,\zeta), i = 1,2$ employs the following equation:

$$f_i^c(\tau,\zeta) = \left\{\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} c_i(u,v)e^{-j2\pi\left(u\tau/M+v\zeta/N\right)} \mid i = 1,2; \tau = 0,1,\cdots,M-1; \zeta = 0,1,\cdots,N-1\right\} \tag{10}$$

where $c_i(u,v), i = 1,2$ denotes the curvatures at $(u,v)$.

**Definition 8.** (Cumulative shape identity) The cumulative functions defined on the intrinsic shape identity given by Equation (9) with the following form

$$A_i^f(r) = \sum_{\tau=0}^{M-1}\sum_{\zeta=0}^{N-1} A(r,\tau,\zeta)\left|f_i^c(\tau,\zeta)\right|, r \in 0,1,\cdots,M/2; M \le N; i = 1,2 \tag{11}$$

where $A(r,\tau,\zeta) = \begin{cases} 1 & r - 0.5 \le \sqrt{\left(\tau - M/2\right)^2 + \left(\zeta - N/2\right)^2} \le r + 0.5 \\ 0 & otherwise \end{cases}$, are called the cumulative shape identities (CSI). $A_1^f(r)$ is called the first cumulative shape identity, and $A_2^f(r)$ the second cumulative shape identity, with respect to first and second principal curvatures of the surface. In addition,

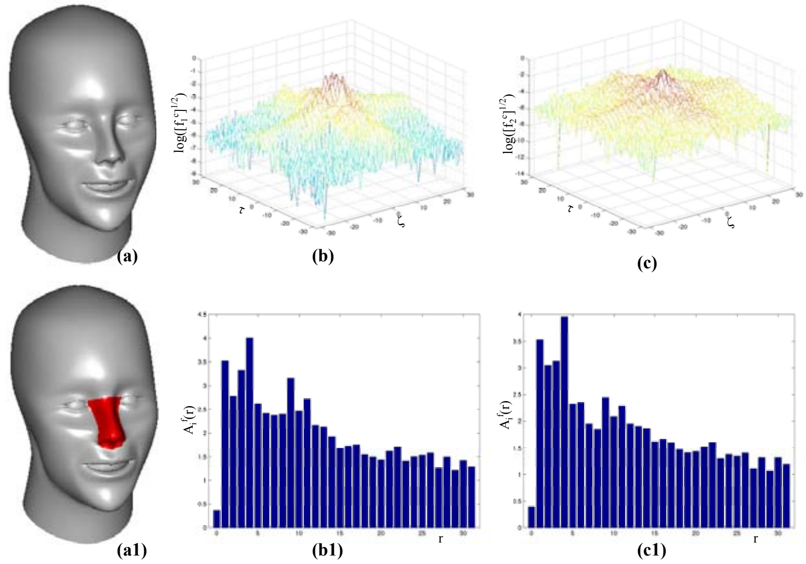

Figure 3. The intrinsic shape identities of a feature and its cumulative identities. (a), (a1), The head with the ROI on the nose as a shape feature; (b), (b1), The first intrinsic identity of the feature and its first cumulative identity; (c), (c1), The second intrinsic identity of the feature and its second cumulative identity.

the ISI is invariant under variations in $(u,v)$ parameterisation. These properties are pre-determined by the characteristics of the curvature invariance.

The computation of the principal curvatures depends on the underlying representation of the model. For instance, if the underlying model data is a point cloud or a mesh model, the calculation of both principal curvatures has to be approximated by using adjacent points or vertices. Feature fitting technique could help to find the equivalent NURBS representation of the surface. Figure 3 depicts an example of the intrinsic shape identities and their corresponding cumulative shape identities of a form feature—the nose.

# 5 Discussions and considerations

Unlike most of the existing featured-based shape knowledge indexing approaches, which analyses shapes by structural feature elements, the method proposed in this paper focuses on local ROI of the FSF. It could not be applied to directly handle structured global shapes.

**Invariance issues.** One of the most important criterions for a shape identity is that it should be invariant under different representations. In the previous sections several shape identities were theoretically proven to possess different level of invariant properties under transformation, starting point, and re-parameterisation. This is because of the intrinsic nature of the cumulative function. For instance, the changing of the start point in NFSI means that the row or the column of the Fourier spectrum $F^S$ may be rearranged. We can easily prove that the rearranged matrix is computationally equivalent to the original one, i.e., $F^{S'} = F^S$, and the NFSI remains unaffected.

**Robustness.** On the other hand, under the input noises of the ROI the shape identity should also behave invariantly. As demonstrated in Figure 4, we choose different ROIs containing the same shape feature, and compare the variation of both the NFSI and the CSI. The results show that the former identity is more stable than the latter in our experiment environment. The reason for the unstable behaviour of the CSI is because of the position variation of the resampled points due to the boundary changing. To apply CSI, the repositioning of the sampling starting point is needed in order to compensate the deviation in CSI introduced by boundary variation. Table 2 gives the result analysis of Figure 4, in which we can observe

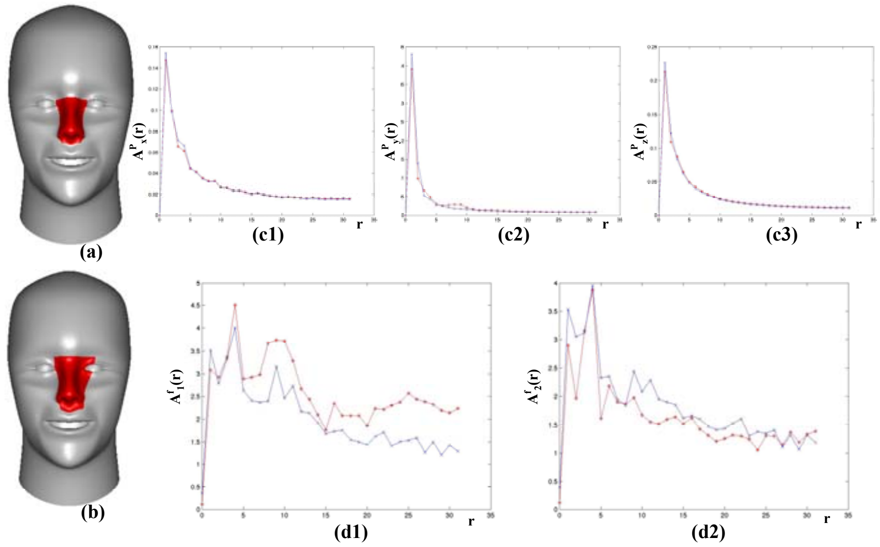


Figure 4. Robustness analysis for the normalized Fourier shape identities and the cumulative shape identities under boundary variation (Blue colour for (a) and red colour for (b)). (a), (b), Different ROIs contain the same shape feature; (c1), (c2), (c3), The variation of the normalized Fourier shape identity in x, y, and z-coordinate components. (d1), (d2), The variation of the first and the second cumulative shape identities.

that the maximum absolute variation for NFSI in y-axis is about 0.04; and 9.2% relative to the value threshold of this component; and for CSI in first curvature is about 1.27 and 28%, respectively.

Identity variations under different input data can be obtained by measuring the norm of the identity in the same category. For instance, the deviation of the cumulative shape identity for Figure 4-(a) and (b) can be measured by $D_i^c = \sum_{r=1}^{M/2} \left| A_{ib}^f(r) - A_{ia}^f(r) \right|, i = 1,2$. The empirical valve-values given according to the experiences of the user can be employed to decide whether feature (a) and (b) are the same or similar to each other. For example, if $D_i^c \le \varepsilon_i, i = 1,2$, where $\varepsilon_i, i = 1,2$ is the given valve-value, then the shapes of the ROIs in Figure 4-(a) and (b) are thought as similar.

Table 2. Data analysis for different identities

| No. | Name of Identity | | Absolute FS value (Max.) | | Relative variation (%) (Max.) |
|---|---|---|---|---|---|
| | | | Value | Variation | |
| 1 | NFSI | x | 0.154249683 | 0.006505415 | 4.217457512 |
| | | y | 0.430999160 | 0.040633142 | 9.427661532 |
| | | z | 0.226709113 | 0.013184860 | 5.815761088 |
| 2 | CSI | $c_1$ | 4.511067867 | 1.271586418 | 28.188146478 |
| | | $c_2$ | 3.953641176 | 1.083031654 | 27.393271318 |

Legend:
FS Fourier Spectrum
NFSI Normalized Fourier Shape Identity
CSI Cumulative Shape Identity

**Simplicity.** Supposing the sampling grid is $M = N = 64$, then the element number in the FSI is $64 \times 64 \times 3$, while that in the NFSI is $32 \times 3$; in the ISI the number of elements is $64 \times 64 \times 2$, and that in the CSI is $32 \times 2$. From FSI to CSI, the representation of the shape identities becomes more and more simple. This dramatic simplification on the representation of the identities can greatly promote the speed of shape knowledge indexing, while saving enormous storage space.

**Adaptability.** Since the identities are defined on the neutral discrete shape distribution, the input data for the calculation of the feature identity can be either from a point cloud model, a mesh model, or a NURBS-based surface model. Due to the input restriction of the 2D DFT, the proposed identity measurements for freeform shape features are exclusively working within the ROI, where the sampling boundary is explicitly defined. However, as a matter of fact, the fundamentals in this paper could be extended to handle the global shape by treating it as a set of form features.

# 6 Conclusions and future work

In this paper, we have presented and evaluated several measurements regarding the identity of FSFs, namely the FSI, the NFSI, the ISI, and the CSI. They could meet with different application requirements. For instance, the NFSI is computationally cheaper than CSI, while the CSI is better than NFSI in storage consumption. All these identity definitions were defined in a unified framework, while avoiding the complexity of the underlying model. To compensate the computational and storage expense, the identity definitions in this paper in turn become more and more simple, providing multiple choices to meet different needs. The experimental results show that the measurements given as the identity of the FSF is robust and applicable. Finally, the results of our work could be utilized to facilitate the shape research and design knowledge processing, especially in industrial design, where the shape aspect is the most concerned.

Future work includes: extensive comparative study concerning existing shape knowledge indexing methods; the applications of the proposed approaches; adaptive sampling strategy for model simplification; indexing scheme for global shape knowledge coding; shape knowledge estimation from partial or sparse Fourier spectrum; methodology for the fusion of shape analysis and shape modeling knowledge.

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