C-K THEORY-BASED FORMULATION OF A GEOMETRIC MODELING APPROACH

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ABSTRACT

This paper presents a novel approach to geometric modeling based on the Concept-Knowledge (C-K) theory of design. By using C-K theoretic formalisms, we create a C-K map that represents the design process, including various Euclidean and fractal shapes in the concept domain and geometric modeling knowledge in the knowledge domain. A key innovation in this respect is the introduction of creative knowledge into the knowledge domain without prior justification or provenance. The injected creative knowledge results in a point cloud creation algorithm that allows for the modeling of shapes suitable for manufacturing. The algorithm has been successfully applied in two case studies: modeling a mechanical object (a gear) and a fractal object (the building block of a fractal called McWorter's Pentigree). These examples demonstrate the algorithm's scalability and effectiveness in generating complex shapes, highlighting the practical utility of the C-K design-theoretic approach in tackling geometric modeling challenges.

Keywords: Geometric Modeling, Fractal, Knowledge, C-K Theory

1 INTRODUCTION

Geometric modeling provides the tools necessary to create, analyze, and manipulate the shapes and structures of objects in a digital environment, playing a fundamental role in Computer-Aided Design (CAD). The digital data produced by CAD are then used in the downstream of product life-cycle for materializing the underlying shape using additive, subtractive manufacturing processes, ensuring that the fabrication accuracy [1]. However, the two primary categories of geometric modeling, Euclidean and fractal, offer distinct approaches to representing shapes. Euclidean geometric modeling focuses on traditional shapes such as lines, circles, and polygons, which are essential in engineering and architectural applications due to their predictable mathematical properties. On the other hand, fractal geometric modeling represents complex, self-similar structures found in nature, such as terrain, clouds, and biological organisms [2,3]. Ullah et al. [4] highlighted the applications of fractals across fields such as manufacturing engineering, architecture, communication engineering, and computer graphics. Fractals also hold significant importance in biomedical engineering, as many natural forms in living organisms display fractal characteristics [4, 5].

A key component in the effectiveness of geometric modeling systems is knowledge representation, which ensures accurate communication and application of geometric principles in manufacturing technologies. In geometric modeling, knowledge serves as both a foundational element and a dynamic resource that informs decision-making, and creative ideas and ensures the relevance and feasibility of design solutions. Knowledge can be described as a statement that aligns with a belief that is both justified and true [6]. Knowledge embedded within the geometric modeling process influences the effectiveness and precision of the resulting models. This includes mathematical principles, material properties, and design constraints, all of which contribute to the accuracy and usability of the models in real-world applications.

Geometric modeling not only provides the technical precision required to visualize, test, and optimize design ideas, but it also integrates seamlessly with design theory, which offers conceptual frameworks and problem-solving methodologies that guide the overall design process. In design theory, knowledge is embodied in design tools, methods, and processes. Therefore, the role of knowledge in design theory

can be implemented in improving geometric modeling. One design theory that can be particularly beneficial in the context of geometric modeling is the C-K theory (Concept-Knowledge theory), which provides a structured framework for innovation and design [7–9]. In this framework, the design process operates within two distinct spaces: the concept space and the knowledge space. As the design process unfolds, both spaces expand concurrently. This expansion relies on existing knowledge while also contributing to the creation of new knowledge, particularly when an innovative or creative design replaces a conventional one within the concept space [9].

This paper introduces a novel approach to geometric modeling that leverages C-K theory to enhance the design process. The proposed methodology is demonstrated through a case study focusing on Euclidean and fractal shapes, where a newly developed algorithm is employed to explore and validate the effectiveness of the approach.

2 KNOWLEDGE AND C-K THEORY IN GEOMETRIC MODELING

Ullah [6] described four types of knowledge: definitional, deductive, inductive, and creative knowledge. Definitional knowledge refers to universally accepted ideas or concepts that are true beyond contradiction. This form of knowledge serves as the foundation for understanding certain phenomena and remains unchallenged within its logical framework. Deductive knowledge, by contrast, is derived through logical reasoning based on definitional knowledge, where relations of ideas are established to infer new conclusions. Inductive knowledge, however, emerges from empirical experience and is obtained through observation of the world. It is based on the logical process of induction, and concludes specific instances to form generalizations, often referred to as "matters of fact." Finally, creative knowledge arises from imaginative and pragmatic activities, where novel ideas or preferences are formulated through innovative processes that transcend existing knowledge structures. The integration of these four types of knowledge results in what Ullah describes as compound knowledge—a synthesis of various knowledge forms that offers a more comprehensive and nuanced understanding of a given topic or problem. Compound knowledge is not merely the sum of its parts, but rather a complex interplay of definitional, deductive, inductive, and creative knowledge that enables deeper insights and more innovative solutions [6].

In the context of design theory, knowledge plays a fundamental role in shaping both the processes and outcomes of design. Design theory provides a structured way to conceptualize and explain the processes involved in the creation of artifacts, whether they be physical (hardware) or digital (software). One prominent design theory that categorizes knowledge and its application in a systematic manner is the C-K theory. C-K theory, short for Concept-Knowledge Theory offers a framework that distinguishes between two interrelated spaces: C-Space (Concept Space) and K-Space (Knowledge Space). The core idea of this theory is that innovation and problem-solving occur through the dynamic interaction between these two spaces. The interaction between C-Space and K-Space is central to the process of innovation. The introduction of a new concept in the C-Space, particularly one that is uncertain or novel, challenges existing knowledge structures in the K-Space. As it is further developed and refined, new knowledge is generated in support of the concept, thus leading to an expansion of the K-Space. This continuous cycle of concept development and knowledge expansion not only fosters innovation but also enhances the design process by allowing for the integration of novel ideas and approaches into established knowledge domains. Therefore, in the C-K theory of design, creative knowledge is crucial [7-9].

Geometric modeling, as a discipline, involves the use of mathematical and computational techniques to represent shapes, objects, and their properties. The creative process in geometric modeling is a structured yet flexible series of stages that allow the modeler to explore and refine geometric forms. This process is inherently iterative, requiring the continuous application of creative thought to optimize the design and representation of geometric structures. Therefore, creative knowledge serves as the foundation for the creative process in geometric modeling. In this regard, creative knowledge is developed, functioning as a key mechanism behind the geometric modeling process. The arrangement of the process is schematically illustrated in Figure 1.

Figure 1. C-K map of the geometric modeling

Figure 1 shows a C-K map of the geometric modeling of both Euclidean and fractal shapes. The C-K map is divided into two interconnected domains: Concept Domain and Knowledge Domain. In the Knowledge Domain, four distinct knowledge areas are identified. Here, K1 represents the knowledge of Euclidean geometric modeling, emphasizing the knowledge required to create Euclidean shapes. Similarly, K2 denotes the knowledge of fractal geometric modeling, a knowledge to utilize specific mapping techniques and algorithms to generate fractal shapes. On the other hand, K3, the knowledge for Design for Manufacturing refers to the knowledge required to create manufacturable shapes, Euclidean or fractal. This knowledge is utilized to apply geometric principles in manufacturing processes. The Knowledge Domain is enriched with another chuck of knowledge denoted as K4, which represents creative knowledge required for design and manufacturing of either of the shapes. This creative knowledge may replace K1, K2, and K3. In the Concept Domain, different types of geometric shapes and their generative processes. Euclidean shapes are derived from the application of K1 (knowledge of design for manufacturing of Euclidean shapes) within the Knowledge Domain, showcasing their foundational role in conventional manufacturing practices. Fractal shapes, characterized by their repetitive and complex structures, are generated through the application of K3 (knowledge of fractal geometric modeling). Further extending the applicability of geometric modeling knowledge, manufacturable shapes represent adaptations of these geometries specifically customized for practical use in manufacturing processes. These manufacturable shapes are the output of K3 (knowledge of design for manufacturing).

In design for manufacturing, topology plays a critical role in as it dictates the relationships within a design, directly influencing the feasibility and efficiency of the manufacturing process. Therefore, knowledge of design for manufacturing (K3) must integrate an understanding of both the geometry of the object and its topology in relation to underlying manufacturing processes. This integration ensures that the design is compatible with the manufacturing method, whether it involves traditional techniques or advanced processes such as additive manufacturing. For example, in 3D printing, the interaction between the topology and the printer head governs not only the precision of the final product but also the efficiency of the tool path. Similarly, in subtractive manufacturing, such as turning or milling, the cutting tool's interaction with the object's topology determines the accuracy and quality of the final product. Thus, the shape and the tool path—essentially, the topology—are critical determining factors. If any issues arise with these aspects, it may impede the manufacturing process. Failure to account for the topology of a design can led to complications during manufacturing, including tool path inefficiencies, structural weaknesses, or even the inability to produce the object.

As illustrated in Figure 1, a creative knowledge of Design for Manufacturing is utilized in the creative process (K4). In this regard, authors proposed a creative-knowledge-based algorithm as a novel contribution to the creative knowledge of Design for Manufacturing, explicitly integrating C-K theory to structure the process of creative knowledge expansion.

Algorithm 1 Analytical Point Clouds Creation Algorithm Define Instantaneous Distance: $r_i \in \mathbb{R}, i = 1, ..., n$ Instantaneous Angle (in degrees) : $\theta_i \in \mathbb{R}, i = 1, ..., n$ Plane: $(u, v) \in \{(x, y), (y, z), (x, z)\}\$ Center Point: $P_c = (P_{cu}, P_{cv}) \in \mathbb{R}^2$

Iterate for $i = 1, \ldots, n$ do Calculate $P_i = (P_{iu}, P_{iv})$ as follows: $P_{iu} = P_{cu} + r_i \cos\left(\frac{\pi}{180}\theta_i\right)$ $P_{iv} = P_{cv} + r_i \sin\left(\frac{\pi}{180}\theta_i\right)$

end for

Output

Analytic Point Cloud: $PC = \{P_i = (P_{iu}, P_{iv}) \mid i = 1, ..., n\}$

In order to use the analytical point clouds creation algorithm, the user needs to understand how to determine the values of the instantaneous distance (r_i) and instantaneous angle (θ_i) . The first step is the definition step, the second step is the iteration step, and the last step is the output step. In the definition step, instantaneous distances ($r_i \in \mathbb{R} \mid i = 1, ..., n$), instantaneous angles (in degrees) ($\theta_i \in \mathbb{R} \mid i = 1, ...,$ *n*), plane $((u,v) \in \{(x,y), (y,z), (x,z)\})$, and center point $(P_c = (P_{cu}, P_{cv}) \in \mathbb{R}^2)$ are defined. In the iteration step, points $P_i = (P_{iu}, P_{iv}), i = 0, 1, ..., n$ is calculated, as follows: $P_{iu} = P_{cu} + r_i \times \cos\left(\frac{\pi}{180}\theta_i\right)$ and $P_{iv} = P_{cv} + r_i \times \sin\left(\frac{\pi}{180}\theta_i\right)$. The last step, i.e., the output step, outputs the resultant $PC = \{P_i = (P_{iu}, P_{iu}\})$. P_{in}) | $i = 1, ..., n$.

For the sake of the better understanding, Figure 2 shows two examples of Algorithm 1-created point clouds. For both cases, $P_c = (10, 10)$ and $(u, v) = (x, y)$ are chosen. (Note that for all cases in the article $(u, v) = (x, y)$ is used.) In the first example (Figure 2(a)), the instantaneous distances are kept constant, i.e., $r_i = 25$, for all ii = 1,…,5. The instantaneous angles are increased linearly. In this case, is chosen. The resultant PC is shown in P_{iy} vs P_{ix} plot. As seen in Figure 2(a), the resultant *PC* represents a pentagon shape. In the other example (Figure 2(b)), similar strategy is used (instantaneous distances are kept constant and instantaneous angles are increased linearly) but the number of points is increased by setting a larger value of *n*, i.e., $n = 25$. This time, the shape represents a circular shape. This way, setting of instantaneous distances, instantaneous angles, coordinate system, and center point, different shapes can be represented by the resultant *PC*. Tashi et al. [10] have used a similar algorithm to create point clouds for modeling artifacts, which are all Euclidian shapes.

Figure 2. Generated point clouds using the Analytical Point Cloud Creation algorithm. (a) Pentagon; (b) Circle.

3 RESULT

This section describes the implementation of a geometric modeling approach involving creative knowledge. Recall the creative-knowledge-based algorithm (Algorithm 1) implemented in the geometric modeling approach in Section 2. A novel geometric modeling approach is applied to create a digital representation of part of a gear object and pentagon fractal. Point clouds generated from the analytical point cloud creation algorithm of a gear object and a pentagon fractal are shown in sub-section below.

3.1 Euclidean shape

In this subsection, Algorithm 1 is applied to generate several Euclidean shapes, which serve as the foundation for manufacturing a gear-shaped object. The process begins by defining the parameters of the algorithm to generate a point cloud representation of the desired shapes. This step ensures that the algorithm can accurately model the geometry of each component of the gear. Once the point cloud is established, the algorithm is applied to construct the final shape, taking into account the specific geometric requirements of every part of the gear. Further explanation of the process is shown in Figure 4 as follows.

Figure 4. The creative process of point cloud creation of gear object.

Figure 4. The creative process of point cloud creation of gear object (continued).

Figure 4 shows the creative process of point cloud creation of gear object. Initially, this process begins with the generation of a point cloud, which is produced through the application of an analytical point cloud creation algorithm. After generating the point cloud, the next step involves processing these points to define a 2D closed boundary that captures the outer edges of the target geometry. This boundary serves as a crucial element in transitioning from a scattered set of points to a more structured representation of the object. Once the boundary has been defined, the resulting 2D geometry can be utilized to construct a surface model, which is the digital representation of the object's surface. This surface model is developed by importing the point cloud data into a commercially available 3D CAD (computer-aided design) system. Within the CAD environment, the point cloud data is processed and converted into a digital model that accurately represents the geometry of the gear. This CAD model can subsequently be utilized for product manufacturing through additive manufacturing techniques, as illustrated for the 3D-printed gear object in Figure 4. The results provided valuable insights into the algorithm's suitability for complex geometries of Euclidean shapes.

3.2 Fractal Shape

In the conventional approach to fractal generation, fractals are generated through the application of a set of mathematical functions in a repetitive, iterative process. This method is known as Iterated Function Systems (IFS) [10]. An IFS involves a finite collection of functions that are applied repeatedly to generate increasingly complex structures. It typically involves a set of functions, each of which defines a transformation (or mapping) that acts on the points of the shape. The transformations in IFS can be contractive mappings, which progressively reduce the distance between points with each iteration, resulting in a more refined and intricate structure as the process advances. In many fractals, the mappings used are affine transformations, which include scaling, rotation, translation, and shearing [10, 11]. The creation process of fractal geometry involves iterative repetition corresponding to the fractal's hierarchical levels. To create manufacturable fractal shapes, several researchers have proposed that the levels of the fractal, or the degree of similarity between the iterations of the fractal, must be meticulously controlled [4]. This is crucial as the inherent complexity and self-similarity of fractals can present challenges when translating them into tangible, manufacturable objects. Consequently, any algorithm designed to generate fractal shapes must possess the capability to control level of the fractal. The algorithm must not only generate the fractal pattern but also offer a way to manipulate key parameters to ensure that the final shape can be effectively realized in a manufacturing environment. As a result, a configuration is proposed to control the level of fractal creation. Using the Analytical Point Cloud Creation algorithm, a process to create fractal level 0 and fractal level 1 is illustrated in Figure 3 as follows.

Figure 3. The process of fractal creation using the Analytical Point Clouds Algorithm.

Figure 3 shows the process of creating fractal level 0 and fractal level 1 using the Analytical Point Cloud Creation Algorithm. As described in Algorithm 1, the first step of the process is to define instantaneous distance $(r_i \in \mathbb{R} \mid i = 1, ..., n)$, instantaneous angle $(\theta_i \in \mathbb{R} \mid i = 1, ..., n)$, and center point $(P_c = (P_{cu},$ P_{cv}) $\in \mathbb{R}^2$). These parameters are used in Algorithm 1 in order to create the point cloud, resulting in P_i $= \{(P_{i1}, P_{i2}) \mid i = 1, ..., n\}$. These points represent the level 0 fractal, i.e., the points represent the base shape from which the subsequent levels of the fractal are generated. For the next repetition of the algorithm, the analytical point cloud (P_i) from fractal level 0 is used as center point, so $P_c = (P_{cu}, P_{cv})$ = P_i , $\exists i \in \{1, ..., n\}$. The same process is repeated by redefining instantaneous distance ($r_i \in \Re \mid i = 1$, ..., *n*), instantaneous angle $(\theta_i \in \mathbb{R} \mid i = 1, ..., n)$ for all P_i , $i = 1, ..., n$. The parameters are processed using the Analytical Point Clouds Algorithm, resulting in analytical point clouds as fractal level 1. The application of this approach is shown in Figure 5 below.

In Figure 5, a proposed approach using the same analytical algorithm is done to model to generate the point cloud data of a pentagon fractal. The steps involved are illustrated in Figure 3 above. The process begins by creating fractal level 0. First, center points $P_c = (10, 10)$ is defined. Subsequently, the parameters r_i and θ_i are specified. In this instance, r_i is held constant where $r_i = 10$, while θ_i increases linearly, with θ_1 =18.5° and $\Delta\theta$ = 72°. The values of r_i and θ_i for this first step are critical to define as it will define the scaling angular rotation for the subsequent levels of the fractal. These parameters then proceed using iterative calculation in Analytical Point Cloud Creation Algorithm, resulting in the formation analytical point cloud $P_c = (P_{cu}, P_{cv})$ as pentagon fractal level 0 as shown in Figure 5. During the next phase, additional pentagons, each with progressively smaller sizes, are generated using an approach illustrated in Figure 3. These smaller pentagons are positioned such that their centers coincide with the vertices of the initial pentagon. The analytical point cloud P_c in fractal level 0 becomes center points for the process of creating fractal level 1 ($P_c = (P_{cu}, P_{cv}) = P_i$, $\exists i \in \{1, ..., n\}$). Furthermore, r_i and θ_i are redefined and used for the iterative calculation in the Analytical Point Cloud Creation algorithm. This process results in pentagon fractal level 1 as shown in Figure 5.

Figure 6. The creative process of point cloud creation of part of a pentagon fractal.

Figure 6 illustrates the creative process of point cloud creation of part of a pentagon fractal. The point clouds of pentagon fractal level 1 are then processed to define a 2D closed boundary around the points. This boundary is essential for creating a structured surface model. The surface model is generated by importing point cloud data into a commercially available 3D computer-aided design (CAD) system. Within this CAD environment, the point cloud data is processed and transformed into a digital model that precisely captures the geometry of the pentagon fractal. This digital model can be utilized for manufacturing the product using additive manufacturing techniques, as demonstrated by the 3D-printed pentagon fractal shown in Figure 6. This method highlights the versatility of the Analytical Point Cloud Creation, demonstrating its applicability in generating complex geometric models.

4 CONCLUDING REMARKS

This paper presents a novel geometric modeling approach that integrates C-K theory (Concept-Knowledge theory) to enhance creative knowledge in design processes using geometric modeling. The study explores the potential of two distinct forms of geometric modeling—Euclidean geometric modeling and fractal geometric modeling. A C-K map is illustrated to show the concept and knowledge required for Euclidean and fractal geometric modeling. In the concept domain, Euclidean shapes, fractal shapes, and manufacturable shapes are shown. In the knowledge domain, knowledge of Euclidean geometric modeling $(K1)$, knowledge of fractal geometric modeling $(K2)$, knowledge of design manufacturing (K4), and creative knowledge of Design for Manufacturing (K4) are shown. The creative knowledge of Design for Manufacturing (K4) is proposed as a replacement for the conventional knowledge typically required in these modeling approaches. In this framework, creative knowledge is formalized through the introduction of a new algorithm, the Analytical Point Cloud Algorithm, which serves as the computational tool for generating analytical point clouds corresponding to the desired shapes. The Analytical Point Cloud Algorithm operates by utilizing a set of parameters, namely center points, instantaneous angle, and instantaneous distance, which guide the point generation process. The points are generated on specifically chosen planes, providing a high degree of flexibility in shaping and modeling intricate and complex forms. This adaptability is crucial in allowing the algorithm to create both Euclidean and fractal-based structures with a creative knowledge foundation, rather than relying solely on traditional geometric principles.

Regardless of whether the shape is Euclidean or fractal, the settings should be kept simple. This implies that there is no need to explicitly write out complex conventional process for the shape and an algorithm that can implicitly represent all shapes is advised. To demonstrate the practical application and effectiveness of this algorithm, a case study is conducted in which the shape of a mechanical object, specifically a gear, is modeled alongside a fractal structure known as McWorter's Pentigree. Through these case studies, the paper illustrates the versatility and potential of the proposed method, showing how it can facilitate the creation of diverse shapes in both traditional and fractal geometric contexts. By developing and applying a creative knowledge-based algorithm, the research successfully demonstrated how analytical point clouds can be used to generate digital models of complex structures. The integration of C-K theory proved effective in structuring the creative process and expanding the knowledge space, leading to the production of more precise and innovative geometric models. This approach shows how creative knowledge can replace traditional knowledge domains to develop versatile and manufacturable shapes, as exemplified through the case studies on gear objects and pentagon fractals.

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